cs302 Theory of Computation	UVa Spring 2008

Notes: Nondeterministic Pushdown Automata

Thursday, 7 February

Upcoming Schedule

Now: Problem Set 2 is due.

Tuesday, 19 February: Problem Set 3 is due. PS3 will be posted before the next class and will cover material through the end of Chapter 2 of the textbook and Class 29 (14 February).

Model of Computation for Deterministic Pushdown Automata

To define the model of computation for a DPDA, we define the extended transition function, δ^*, similarly to how we did for DFAs, except we need to model the stack.

Recall that the transition function is:

δ: $Q \times \Sigma \times \Gamma_\epsilon \rightarrow Q \times \Gamma_\epsilon$

What is the type of the extended transition function of at DPDA, δ^*:

[ ]

As with DFAs, we can define δ^* for all possible inputs using induction on the input string. But, we need to be careful to consider all cases for the stack transitions.

$\delta^*(q, \epsilon, s) = (q, s)$
For all $a \in \Sigma, x \in \Sigma^*, \gamma \in \Gamma^*$: $\delta^*(q, ax, \gamma) =$

1. if $(q_t, \epsilon) \in \delta(q, a, \epsilon)$: [ ]
2. $\forall h_i \in \Gamma_\epsilon$, if [ ] $\wedge\, \gamma = push(h, \gamma_r)$:

   $$\delta^*(q, ax, push(h, \gamma_r)) = (q_t, push(h_i, \gamma_r))$$
3. What is missing? (left as exercise for PS3)

Accepting State Model: A deterministic pushdown automata, $A = (Q, \Sigma, \Gamma, \delta, q_0, F)$ accepts a string $w \in \Sigma^*$ if and only if:

$$\delta^*(q_0, w, []) \rightarrow (q_f, s) \text{ and } q_f \in F$$

Weak Empty Stack Model: A deterministic pushdown automata, $A = (Q, \Sigma, \Gamma, \delta, q_0)$ (note there is no F now) accepts a string $w \in \Sigma^*$ if and only if:

$$\delta^*(q_0, w, []) \rightarrow (q, s) \text{ and } s = \epsilon$$

Can all languages that can be accepted by the *accepting state model* be accepted by the *weak empty stack model*?

Empty Stack Model: A deterministic pushdown automata, $A = (Q, \Sigma, \Gamma, \delta, q_0, Z_0)$ accepts a string $w \in \Sigma^*$ if and only if:

$$\delta^*(q_0, w, Z_0) \rightarrow (q, s) \text{ and } s = \epsilon$$

Challenge question: is the set of languages that can be recognized by a DPDA under the accepting state model equivalent to the set of languages that can be recognized by a DPDA under the empty stack model?

Nondeterministic Pushdown Automaton

A *nondeterministic pushdown automaton* (this is what Sipser calls a *pushdown automaton*) is a 6-tuple $(Q, \Sigma, \Gamma, \delta, q_0, F)$ where $Q, \Sigma, \Gamma, q_0, F$ are defined as they are for DPDA and the transition function is defined:

$$\delta : Q \times \Sigma_\epsilon \times \Gamma_\epsilon \rightarrow \boxed{}$$

Example. Define a NPDA that recognizes the language $\{ww | w \in \Sigma^*\}$.