

Quadratic Equations have the form $ax^2 + bx + c = 0$, where $a \neq 0$.

Methods for solving quadratic equations:

- factoring
- taking square roots
- completing the square
- using the quadratic formula

Solve by factoring:

1. Move everything to one side of the equation and be sure to make the x^2 term is positive.

It will have the form $ax^2 + bx + c = 0$

2. Factor the left-hand side of the equation.
3. Set each factor equal to zero.
4. Solve each of the resulting equations.

Example 1: Solve the equation.

$$x^2 - 3x = 4$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$x-4=0 \quad x+1=0$$

$$x=4 \quad x=-1$$

Example 2: Solve the equation.

$$-3x^2 = -3x - 60$$

$$\begin{aligned} -3x^2 + 3x + 60 &= 0 \\ -3(x^2 - x - 20) &= 0 \\ \frac{-3}{-3} &\quad \frac{0}{-3} \\ x^2 - x - 20 &= 0 \\ (x-5)(x+4) &= 0 \end{aligned}$$

$$\begin{array}{ll} x-5=0 & x+4=0 \\ x=5 & x=-4 \end{array}$$

Example 3: Solve the equation.

$$3x^2 + 5x + 2 = 0$$

$$\begin{aligned} (3x+2)(x+1) &= 0 \\ 3x+2=0 & \quad x+1=0 \\ \frac{3x}{3} = -\frac{2}{3} & \quad x = -1 \\ x = -\frac{2}{3} & \end{aligned}$$

$$\left\{ x = -1, x = -\frac{2}{3} \right\}$$

Example 4: Solve the equation.

$$3x^2 - 10x - 8 = 0$$

$$\begin{array}{l} 1 \cdot 8 \\ 2 \cdot 4 \end{array} \quad \begin{aligned} (3x+2)(x-4) &= 0 \\ 3x+2=0 & \quad x-4=0 \\ \frac{3x}{3} = -\frac{2}{3} & \quad x=4 \\ x = -\frac{2}{3} & \end{aligned}$$

Square Root Method

Equation has the form of $ax^2 - b = 0$, where a and b have to be perfect squares at least in this course.

Example 5: Solve the equation.

$$\begin{array}{r} x^2 - 25 = 0 \\ +25 \quad +25 \\ \hline x^2 = 25 \\ \sqrt{x^2} = \pm \sqrt{25} \\ x = \pm 5 \end{array}$$

Example 6: Solve the equation.

$$\begin{array}{r} 16x^2 - 25 = 0 \\ +25 \quad +25 \\ \hline 16x^2 = \frac{25}{16} \\ \frac{16x^2}{16} = \frac{25}{16} \end{array} \quad \begin{array}{r} x^2 = \frac{25}{16} \\ \sqrt{x^2} = \pm \sqrt{\frac{25}{16}} \\ x = \pm \frac{5}{4} \end{array}$$

Solving by Completing the Square

1. Write the equation in the form $x^2 + bx = c$.
2. Add $\left(\frac{b}{2}\right)^2$ to both sides.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$$

3. Factor the left-hand side.
4. Solve by the square root method.

Example 7: Solve the equation.

$$\begin{array}{r} x^2 - 6x = 13 \\ x^2 - 6x + \left(\frac{6}{2}\right)^2 = 13 + \left(\frac{6}{2}\right)^2 \\ x^2 - 6x + 9 = 13 + 9 \\ (x-3)(x-3) = 22 \\ (x-3)^2 = 22 \end{array} \quad \begin{array}{r} \sqrt{(x-3)^2} = \pm \sqrt{22} \\ x-3 = \pm \sqrt{22} \\ x = 3 \pm \sqrt{22} \\ \left. \begin{array}{l} x = 3 + \sqrt{22}, \\ x = 3 - \sqrt{22} \end{array} \right\} \end{array}$$