Lesson 20: The Family of Quadratic Functions

(Cover 5.5)	**Read:** Section 6.1
Announce: EXAM 2 is Tuesday, March 22 at 6 pm	**Do:** WebWork, Team Homework

The most important points and skills for §5.5

- Students are able to recognize formulas for quadratic functions in standard ($y = ax^2+bx+c$), vertex ($y = a(x-h)^2 + k$) and factored ($y = a(x-r_1)(x-r_2)$) forms and use these forms to determine information about the graph of the function (y–intercept from standard form, vertex from vertex form, and x–intercept(s) from factored form).
- Given a graph of a quadratic function $f(x)$ that shows the locations of two distinct x–intercepts and the coordinates of one other point on the graph, students are able to find an explicit formula for $f(x)$ using factored form.
- Given a graph of a quadratic function $f(x)$ that shows the coordinates of the vertex and one other point on the graph, students are able to find an explicit formula for $f(x)$ using vertex form.
- Given an explicit formula for a quadratic function, students are able to determine the x– and y–coordinates of the vertex as well as the formula for the axis of symmetry of the parabola by completing the square and then noting the values of h and k that are present in the vertex form $y = a(x-h)^2 + k$.
- Students should be able to determine the maximum/minimum of a quadratic function by converting the formula into vertex form and interpreting the meaning of the x– and y–coordinates of the vertex in terms of the "real-world" context described in word problems.

Comment:
Most students are already very familiar with quadratic functions, standard form, and factoring (and we have already worked with these this semester); however, completing the square is quite difficult for many students. When you do examples of completing the square, avoid the temptation to cut corners and *always* describe your calculation methods IN FULL and *put all steps in your boardwork.*

Lesson 21 Supplement:
There is a handout that you may choose to use with this lesson. It is a separate document on the instructors' website.

Suggested Lesson Plan:

00–25 Use this time for a quiz or to do more examples from Sections 5.3 and/or 5.4. **Section 5.3 #25 on page 217** and **Section 5.4 #23 on page 224** are good problems to serve as review. §5.3 #25 emphasizes verbal interpretation of functions. Problems like §5.4 #23 can be difficult for students who often have trouble marking these types of output values on a graph.

Summarize the last lesson in 1-2 sentences. Outline todays lesson.

25–35 Begin the new material with a mini-review of what we've learned about quadratic functions so far. Remind them about the standard equation ($y = ax^2 + bx + c$). This is also a good time to remind them about factored form ($y = a(x - r_1)(x - r_2)$, where r_1 and r_2 are zeros) and what it means to know the zeros of a function.
Introduce the third way of writing a quadratic function: vertex form ($y = a(x - h)^2 + k$). Make sure to define the vertex and axis of symmetry as well as how these may be found in an equation in vertex form. (Note: The supplement for this lesson is a handout that has been used in previous terms regarding vertex form and how it can be interpreted using function transformations. Feel free to use it or distribute to students if you would like.)

Point out the key features of a graph that can be easily seen in each form (y–intercept in standard form, x–intercept(s) in factored form, and vertex in vertex form).

35–45 When you have established all of the terms, have the students practice finding a formula for a quadratic function when given the graph. Give the students two examples to try in their groups. In the first example, you could start with a graph that shows two x–intercepts and the coordinates of one other point (such as **Section 5.5 #13 on page 231**). For the second example, you could start with a graph that shows the coordinates of the vertex and the coordinates of one other point on the graph (such as **Section 5.5 #9 on page 231**). Circulate as the groups work. Some questions that you might find helpful to ask the students are:

- Based on the information that you can see on the graph, which way of writing a quadratic function would be easiest to use here?
- How do the points you can see on the graph help you to figure out the numbers that need to go into the formula?
- What added information do we gain from the extra point (i.e. not the zeros or vertex) on the graph?
- How can you know the sign of the leading coefficient just by looking at the graph?

Generally, students understand how to use zeros and/or vertex points in equations, but often have a difficult time using the "extra" point to solve for the leading coefficient a.

45–60 Move into a graphical discussion of quadratic equations by noting that all quadratic equations are transformations of the function $f(x) = x^2$. Write the two equations $f(x) = x^2$ and $g(x) = 3(x + 7)^2 - 4$ on the board. Ask students to *briefly* discuss in their groups how the graph of $g(x)$ is related to the graph of $f(x)$ using terminology for transformations, and ask them to write $g(x)$ in terms of $f(x)$. Next, write the equation $h(x) = 2x^2 - 8x + 3$ on the board. Note that because $h(x)$ is quadratic, the graph of $h(x)$ should be related to the graph of $f(x)$ through various transformations. You can use this opportunity to note that in standard form, it is harder to determine how the graph of $h(x)$ is related to $f(x)$ than it was to tell how $g(x)$ is related to $f(x)$. Segue into a mini-lecture about completing the square and put $h(x)$ into vertex form.

Note: The discussion of completing the square in the Chapter 5 Tools section (page 239) of the textbook gives a formula, but we don't want to emphasize this. Rather, we want the students to learn an algorithmic approach such as the one used in Examples 1 and 2 (pages 239-240; also Example 2 from pages 227-228).

If you really want to break down the technique into bite-sized pieces, here is one procedure that you could follow (for $h(x) = 2x^2 - 8x + 3$).

Step 1: Factor the leading coefficient out of every term in the function.

$$h(x) = 2[x^2 - 4x + 3/2]$$

Step 2: Look at the number preceding the x–term. Divide this number by 2 and then square that value. number = -4 half the number = -2 which when squared is $(-2)^2 = 4$.

Step 3: Add and subract the value you computed in Step 2 in between the x–term and constant term.

$$h(x) = 2[x^2 - 4x + \mathbf{4} - \mathbf{4} + 3/2]$$

Step 4: Group together the first three terms to have a perfect square.

$$h(x) = 2[(x^2 - 4x + 4) - 4 + 3/2] = 2[(x-2)^2 - 4 + 3/2]$$

Step 5: Combine the constant terms left over outside the perfect square.

$$h(x) = 2[(x-2)^2 - 5/2]$$

Step 6: Distribute the coefficient you factored out in Step 1. $h(x) = 2(x-2)^2 - 5$.

The most common mistakes are (i) not factoring the leading coefficient a out of everything in Step 1 and (ii) not distributing a correctly in Step 6.

There are certainly other algorithms. One of these is the following:

<u>Alternate method for completing the square</u>
Shylynn Lofton, Fall 2009

Let $f(x) = -10x^2 + 20x + 97$. Since the leading coefficient is not 1, factor it out of the first two terms, only. We do this because we are interested in forcing the group of variable expressions into a perfect square. The constant term is not altered in any way. I instruct my students to leave some space inside and outside of the parentheses. The space inside is used to 'complete the square.' The outside space is used to ensure that the expression is balanced.

$$f(x) = -10x^2 + 20x + 97$$
$$= -10(x^2 - 2x \quad) \quad + 97$$
$$= -10(x^2 - 2x + 1) + 10 + 97$$
$$= -10(x^2 - 2x + 1) + 10 + 97$$
$$= -10(x-1)^2 + 107$$

We complete the square by adding 1 to the 'grouping.' But in order to ensure that the value of the expression remains the same, we have to remember that we initially factored out –10. So, although we added 1 to the 'grouping,' we really added –10 to the expression. To balance that addition of –10, we must add +10. This is the point where errors could occur. I constantly remind my students to do a quick mental check of the expression's value. I also remind them to think about the steps that led up to this point. With practice, most students have no problems with this method.