Lesson 9: Introduction to Quadratic Functions

(Cover 3.1)	**Read:** Section 3.2 & "Completing the Square"
Announce/Remind: Gateway, Gateway, Gateway, Gateway, Gateway, Gateway, Gateway!	**Do:** WebWork, Team Homework, **Gateway**

Midterm 1 - Tuesday, February 7, 6-7:30 pm (Note: NOT Michigan time) Location: East Hall 1324
Material to be covered: Sections 1.1-1.5, 2.1-2.5, 3.1-3.2, 4.1-4.2

The most important points and skills for §3.1

- Students should know that the formula for a quadratic equation resembles $y = ax^2 + bx + c$ where $a \neq 0$.
- Given the equation of a quadratic function, students should be able to predict whether the graph is concave up or concave down based on the sign (+ or −) of the leading coefficient, a.
- If students have a quadratic equation and are given an x-value, then they should be able to evaluate the quadratic equation at that x-value, respecting the order of operations.
- Students should know that "solving" a quadratic equation of the form $ax^2 + bx + c = 0$ is means to find the "zeros" of the function $f(x) = ax^2 + bx + c$, that is, all the values of x that can be plugged into the expression on the left to produce an output value of zero. They should also realize that the zeros are the x-values at which the graph of $y = ax^2 + bx + c$ crosses the x-axis.
- Given a quadratic equation of the form $ax^2 + bx + c = 0$, students should be able to find all solutions by factoring into the form $a(x - r)(x - s) = 0$ or by substituting the values of a, b, and c into the quadratic formula
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$
- Given the zeros and vertical intercept of a quadratic function, students should be able to find a formula for the function.

Suggested Lesson Plan:

0-20 Use this time for a quiz and/or to return graded work (quizzes or team homework). Summarize the previous lesson in one or two sentences. If you are not giving a quiz and did not have time to do so last time, have the students try **Section 2.5 Problems #13 and #17 from pages 95-96**. Call on groups to put the graphs on the board and ask them to explain the shape. Make sure they label their axes with an appropriate scale and units.

20-30 Introduce quadratic functions and their general form (standard form $y = ax^2 + bx + c$ with $a \neq 0$). In order to emphasize what quadratic functions are (and that they can be "disguised"), have the students discuss **Section 3.1 Problems #3, #6, and #7 on page 109** in their groups. Then bring them together and make sure that the class has come to a consensus (and that the reasoning is valid).

30-40 Students are generally familiar with quadratic functions, but they are not necessarily truly comfortable with them. (As usual) focus on the Rule of Four rather than always providing

formulas. Point out that an important characteristic of a quadratic function is that the average rate of change over equal intervals is *not* constant (which was the case for linear functions), but is instead always changing. (This ties in well with your discussion of concavity from the previous lesson.)

Begin with a table of values for a function like $f(x) = 3x^2+5x-2$ (or whatever function you plan to continue working with below). Look at properties of the function (increasing vs decreasing, concave up vs concave down) based only on the table. Don't provide the equation until after completing your investigation of the table.

40-55 Then define what it means to find the *zeros* of a function. Note the fact that a quadratic function may have 0, 1, or 2 zeros. Take an example of a quadratic function that factors easily such as $y = 3x^2 + 5x - 2$. Have the students graph the function, and point out that the zeros of the function correspond to the x-intercepts of the graph. Ask the students to factor the function into $y = (3x - 1)(x + 2)$. Make sure they understand how to algebraically solve for the zeros by using the factored form, and make sure their answers correlate with the zeros on their graph.

Wrap up this problem by introducing (reminding students of) the quadratic formula (which can be found on pages 105 and 123). Have the students confirm the answers they just found above by using the quadratic formula to find the zeros of $y = 3x^2 + 5x - 2$. Do NOT spend time in class *deriving* the quadratic formula.

To reinforce these ideas, have the students do **Section 3.1 Problems #10 and #26 on page 109**.

55-65 Have the students practice using the factored form to find a formula for a quadratic function when given the zeros. Give the students two examples to try in their groups such as **Problems #23 and #29 on page 109**. Generally, students understand how to use zeros in equations, but often have a difficult time using the "extra" point to solve for the leading coefficient a.

65-80 The study of quadratic functions has many applications to physics. It is important to consider an example of an object falling under the effect of gravity. Consider a ball (or some more interesting object if you prefer) that is thrown upward from a bridge and is allowed to fall past the bridge all the way to the ground. For example, let $h(t) = -16t^2 + 42t + 120$ denote the height of the ball in feet above the ground t seconds after being released. Make sure that the students understand that *the shape of the height graph does not represent the path of the ball, which is straight up and down.* (This is a common misconception.) Have them work in groups to answer the following questions:

1) How high is the ball when it is released?
2) *Exactly* when does the ball hit the ground? (Use the quadratic formula; are both answers valid?)
3) Sketch a graph of the function h. What is a meaningful domain and range for this problem? (Note that the physical constraints of the situation restrict the domain and range to non-negative values.)
4) Solve $h(t) = 100$. Interpret the solution and illustrate it on your graph.

If you have additional time, you can move on to an example of "Completing the Square" as this is a skill that we will use in the next class. (Consider the Exercises on page 128 as possible examples.)

When you are finished, summarize the key ideas from todays lesson. Before the class ends, remind the student's once again that the **Gateway deadline is imminent!**