CompSci 102 Discrete Mathematics for CS
Spring 2005 Forbes

In-class Questions 5

These questions will also serve as *part of* the warmup for recitation 7

1. Suppose that next June, the Durham Bulls play at leats 1 game a day, but *leq*45 games total. Show that there must be some sequence of days in June during which they play *exactly* 14 games.

2. An inventory list contains a list of 80 items, each marked "available" or "unavailable." There are 50 available items. Show that there are at least two unavailable items in the list either three or six items apart.

3. We want show that if X is any $(n+2)$ element subset of $\{1, 2, \ldots, 2n+1\}$ and m is the greatest element in X, there exist distinct i and j in X with $m = i + j$.

For each element $k \in X - \{m\}$, let

$$a_k = \begin{cases} k & \text{if} k \leq \frac{m}{2} \\ m - k & \text{if} k > \frac{m}{2} \end{cases}$$

(a) How many elements are in the domain of a?

(b) Show that the range of a is contained in $\{1, 2, 3 \ldots, n\}$.

(c) Explain why the previous two results imply that $a_i = a_j$ for some $i \neq j$.

(d) Explain why the previous result implies that there exist distinct i and j in X with $m = i + j$.

4. How many bit strings of length 10 contain

   (a) exactly four 1s?

   (b) at most four 1s?

   (c) at least four 1s?

   (d) an equal number of 0s and 1s?