PRACTICE QUESTIONS AND PROCEDURE for **Quiz #9**

Here is the basic format of the first three parts of the upcoming quiz dealing with tautological and FO relations, along with a few practice questions and some explanation of a procedure for tackling these parts of the quiz.

Part 1: For each of the following sentences, convert the sentence into both *First-Order Checking Form* and *Truth-Functional Form*. (You can use capital letters P, Q, R, etc. for First-Order Checking Form and capital letters A, B, C, etc. for Truth-Functional Form.) Then, state whether the sentence is . . .

(1) not a logical truth,
(2) a logical truth,
(3) more specifically, a first-order validity, or
(4) even more specifically, a tautology.

There's a **basic procedure** to follow for answering this sort of question:

First, since I'm asking you for these questions to write out both the FO Checking Form and the Truth-Functional Form for each sentence, you might as well start by doing just that. Remember that to convert a sentence of FOL into **FO Checking Form**, you 'blind' yourself to every predicate in the sentence other than the identity (=) predicate. So every 'Cube', 'Smaller', 'SameSize', etc. gets replaced by a capital letter like P, Q, or R. But the arguments of those predicates (i.e., the individual constants and the variables), as well as everything else in the sentence (quantifiers, Boolean connectives) remain. (See the examples below and in the lecture notes for lesson 30.) Then, to convert the sentence into **Truth-Functional Form**, you need to put on more drastic 'blinders': replace every atomic sentence (i.e., predicate + its arguments) and every quantified phrase (i.e., quantifier + everything within the scope of the quantifier) with a capital letter like A, B, or C. (Again, there are examples below and in the notes.)

Once you've written out the above conversions, it's time to read and think about the sentence of FOL itself, first just as it's originally presented—*without* any 'blinders' on. Considering the meaning of the sentence, can you think of any way that the sentence could ever be *false* in some world? If you *can*, then it must be merely a *logical possibility*, and you know that it is "**not** a logical truth" (i.e., **sentence type 1**). If, on the other hand, you convince yourself that this sentence could never be false, then you know that it will be one of the sentence types 2-4 (i.e., the three varieties of logical truths). In that case, you'll need to move on to the next step in order to narrow down which type of logical truth it is.

Next step (i.e., for those sentences that you believe to be logical truths): Look at the **FO Checking Form** of the sentence and ask yourself whether, with those particular 'blinders' on, you can still see that the sentence must always be true. If in FO Checking Form you can *no longer see that it is a logical truth* (i.e., the necessity of the sentence has 'evaporated' with your FO-Checking-Form blinders on), then you know that this sentence is of **sentence type 2** (i.e., a logical truth that is *not* more specifically an FO validity or tautology). If, on the other hand, the

necessity of the sentence is still apparent even in FO Checking Form, then you know that you are dealing with one of the sentence types 3-4, and you'll need to continue on to the final step below.

Finally, look at the **Truth Functional Form** of the sentence. If you could see the necessity of the sentence when it was in FO Checking Form but now you can *no longer see that necessity in the Truth Functional Form of the sentence*, then you know that the sentence is of **sentence type 3** (i.e., it's an FO validity that is *not* more specifically a tautology). If, on the other hand, you can *still see even in its Truth Functional Form that the sentence must always be true* (i.e., even when you've blinded yourself to everything except how the Boolean connectives structure the sentence), then you know that you're dealing with **sentence type 4** (i.e., a tautology).

	First-Order Checking Form	**Truth-Funct. Form**
1. ∀x x=x ∧ ∀y y=y	∀x x=x ∧ ∀y y=y	A ∧ B

Sentence type? 3

The quantifiers and the identity predicate are preserved in FO Checking Form, and you can still see the necessity involved. Thus, this sentence is a FO validity. It is *not* more specifically a tautology, however, as seen when you convert it to Truth-Functional Form. There, each of the entire quantified phrases is replaced by a capital letter, and the necessity is no longer visible.

2. SameShape(a,b) → SameShape(b,a)	P(a,b) → P(b,a)	A → B

Sentence type? 2

You can (hopefully) see that the original sentence is a logical truth, but without knowing what the 'blinder' predicate **P** means in the FO Checking Form, you can no longer tell in that form whether the objects *b* and *a* are in a necessarily symmetrical relation to each other, so this sentence is a logical truth but *not* a FO validity (and therefore not a tautology either).

Part 2: For each of the following arguments, state whether the conclusion of the argument is

(1) not a logical consequence of the premises (i.e., the argument is invalid),
(2) a logical consequence of the premises,
(3) more specifically, a first-order consequence of the premises, or
(4) even more specifically, a tautological consequence of the premises.

If you answered (1), then in the space provided beside the argument *draw a counterexample world* that proves the argument invalid. Be sure everything in your world is clearly marked and unambiguous, and that it is specific enough to truly provide a counterexample to the argument.

If you answered (2), then in the space provided beside the argument *convert the entire argument into FO Checking Form* (for your 'blinders' use *alternative predicates*, not capital letters) so as to clearly demonstrate that the consequence relation is no longer visible in FO Checking Form.

If you answered (3), then in the space provided beside the argument *convert the entire argument into BOTH FO Checking Form and Truth-Functional Form* (label which is which) in order to demonstrate that the consequence relation is still visible in FO Checking Form but NOT visible in Truth-Functional Form. (Again, use alternative predicates for FO Checking Form here, but use capital letters A, B, C, etc. for Truth-Functional Form.)

If you answered (4), then in the space provided beside the argument *convert the entire argument into BOTH FO Checking Form and Truth-Functional Form* in order to demonstrate that the consequence relation is still visible in BOTH forms.

Again, there's a basic procedure for tackling this sort of question. Actually, the task instructions above mirror the procedure you should follow (which also closely mirrors the procedure I gave you above for tackling problems in Part 1).

First, consider the argument without any blinders on. If you can think of a scenario in which the premises of the argument could all be *true* but the conclusion *false*, then you've thought of a counterexample world and have proven that the argument is invalid. In that case, the **"type of consequence relation"** is **type 1** (i.e., the conclusion is *not* a logical consequence of the premises), and you should draw out the **counterexample world** to prove it. If, on the other hand, you convince yourself that the argument is valid, that means the conclusion of the argument is a logical consequence of the premises, which means that one of the types of consequence relation 2-4 holds, and you'll need to continue on to the next step.

If the argument is indeed valid, you need to decide which type of consequence obtains. Convert the entire argument into **FO Checking Form** using the ***random predicate method*** (see examples below) and then ask yourself whether—with these FO-Checking-Form blinders on—you can still see that the argument is valid. If you can no longer see the logical consequence relation (i.e., without knowing now the meaning of the blinded-out predicates), then you've demonstrated that the logical consequence in the original argument is a case of **consequence type 2** (i.e., simple logical consequence that is *not* more specifically a case of FO consequence or tautological consequence). If, on the other hand, you can still see the validity of the argument