Rank

Rank of an element is its position in ascending key order.

[2,6,7,8,10,15,18,20,25,30,35,40]

rank(2) = 0

rank(15) = 5

rank(20) = 7

Selection Problem

- Given n unsorted elements, determine the k'th smallest element. That is, determine the element whose rank is k-1.
- Applications
  - Median score on a test.
    - k = ceil(n/2).
  - Median salary of Computer Scientists.
  - Identify people whose salary is in the bottom 10%. First find salary at the 10% rank.

Selection By Sorting

- Sort the n elements.
- Pick up the element with desired rank.
- O(n log n) time.

Divide-And-Conquer Selection

- Small instance has n <= 1. Selection is easy.
- When n > 1, select a pivot element from out of the n elements.
- Partition the n elements into 3 groups left, middle and right as is done in quick sort.
- The rank of the pivot is the location of the pivot following the partitioning.
- If k-1 = rank(pivot), pivot is the desired element.
- If k-1 < rank(pivot), determine the k'th smallest element in left.
- If k-1 > rank(pivot), determine the (k-rank(pivot)-1)'th smallest element in right.

D&C Selection Example

Find kth element of:

a	3	2	8	0	11	10	1	2	9	7	1

Use 3 as the pivot and partition.

a	1	2	1	0	2	3	10	11	9	7	8

rank(pivot) = 5. So pivot is the 6'th smallest element.

D&C Selection Example

a	1	2	1	0	2	3	10	11	9	7	8

- If k = 6 (k-1 = rank(pivot)), pivot is the element we seek.
- If k < 6 (k-1 < rank(pivot)), find k'th smallest element in left partition.
- If k > 6 (k-1 > rank(pivot)), find (k-rank(pivot)-1)'th smallest element in right partition.