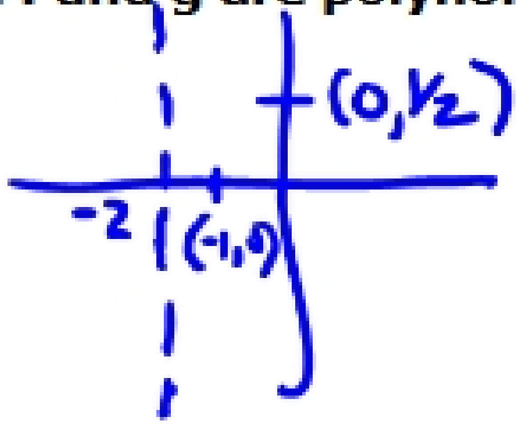
**Definition**: **A rational function is a function that can be written in the form $R(x) = \frac{f(x)}{g(x)}$, where f and g are polynomials.**

**Example 1:**

$R(x) = \frac{x+1}{x+2}$ $f(x)$ $g(x)$




xint $y=0$

$\frac{x+1}{x+2} = 0$

$x+1=0$

$x=-1$ $(-1,0)$

yint $x=0$

$\frac{0+1}{0+2} = \frac{1}{2}$

$(0,\frac{1}{2})$

$x+2=0$

$x=-2 \rightarrow$ vertical asymptote

**The domain of the rational function $R(x) = \frac{f(x)}{g(x)}$ consists of all real numbers x such that $g(x) \neq 0$.**

**Graphs of Rational Functions**

**The first step in graphing a rational function is the x and y intercept.**

**Example 2:**

**Find the x and y intercepts of the following rational function.**

ⓐ $f(x) = \frac{x+3}{x^2-4}$

xint $y=0$

$(x^2-4)\frac{x+3}{x^2-4} = 0\,(x^2-4)$

$x+3=0$

$x=-3$ $(-3,0)$

yint $x=0$

$\frac{0+3}{(0)^2-4} = -\frac{3}{4}$

$(0,-\frac{3}{4})$

ⓑ $\frac{2}{x-1}$

xint $y=0$

$\frac{2}{x-1} = 0$

$2=0$ false

no x intercept

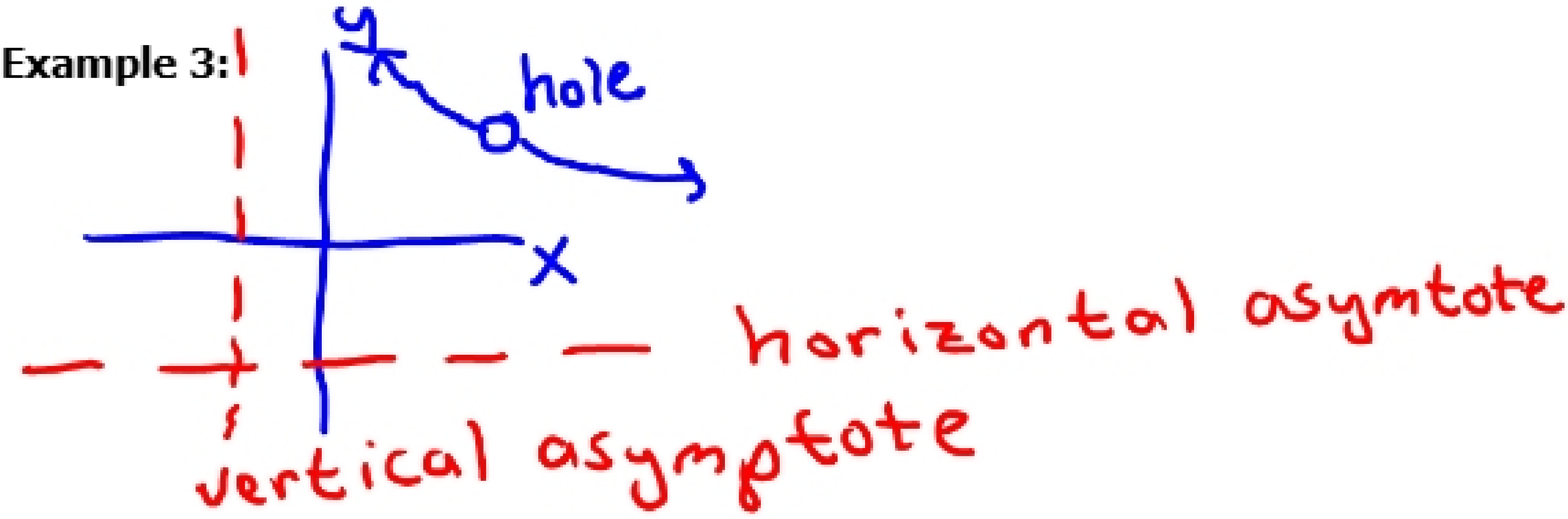
yint $x=0$

$\frac{2}{0-1} = -2$

$(0,-2)$

**The graphs of rational functions are characterized by asymptotes and/or holes.**

**Example 3:**




**<u>To find vertical asymptotes and holes:</u>**

**Factor the numerator and denominator. Look at each factor in the denominator.**

- **If a factor "cancels" with a factor in the numerator, then there is a hole where that factor equals zero.**

- **If a factor doesn't "cancel", then there is a vertical asymptote where that factor equals zero.**

**Example 4:**

**Find any vertical asymptotes or holes for** $\mathbf{f(x) = \dfrac{x^2 - 3x - 10}{x^2 - x - 6}}$.

$$f(x) = \frac{(x+2)(x-5)}{(x+2)(x-3)}$$

hole: $x+2=0$
$x=-2$

VA: $(x-3)=0$
$x=3$

**Example 5:**

**Find any vertical asymptotes or holes for** $f(x) = \frac{x^2 + x - 20}{x - 4}$

$f(x) = \frac{(x+5)(x-4)}{(x-4)}$

hole: $x-4=0$
$x=4$

no vertical asymptote

**Finding Horizontal Asymptotes: of** $R(x) = \frac{f(x)}{g(x)}$

**<u>Case 1</u>: deg(f)<deg(g): horizontal asymptote is** $y = 0$ **(the x-axis).**

**Example 6:** $f(x) = \frac{x^2 + 4x}{5x - x^3} = \frac{x^2+4x}{-x^3+5x}$ $\frac{x^2}{-x^3}$ deg top < deg bottom

HA: $y=0$

**<u>Case 2</u>: deg(f)=deg(g): horizontal asymptote is** $y = \frac{a}{b}$**, where**

**Example7:** $g(x) = \frac{3x^2 + 5x}{4x^2 - 1}$ HA: $y = 3/4$

- **a is leading coefficient of numerator.**
- **b is leading coefficient of denominator.**

**<u>Case 3</u>: deg(f)>deg(g): no horizontal asymptote.**

**Example 8:** $R(x) = \frac{x^9 + 3x^2}{x^6 + 4x^5 - 5}$ $\frac{x^9}{x^6}$ deg top > deg bottom

no HA