<u>Definition</u>: A *rational* function is a function that can be written in the form $R(x) = \frac{f(x)}{g(x)}$, where f and g are polynomials.

Example 1:

The <u>domain</u> of the rational function $R(x) = \frac{f(x)}{g(x)}$ consists of all real numbers x such that $g(x) \neq 0$.

<u>Graphs of Rational Functions</u>

The first step in graphing a rational function is the x and y intercept.

Example 2:

Find the x and y intercepts of the following rational function.

$$\mathbf{f(x) = \frac{x + 3}{x^2 - 4}}$$

The graphs of rational functions are characterized by asymptotes and/or holes.

Example 3:

<u>To find vertical asymptotes and holes:</u>

Factor the numerator and denominator. Look at each factor in the denominator.

- If a factor "cancels" with a factor in the numerator, then there is a hole where that factor equals zero.
- If a factor doesn't "cancel", then there is a vertical asymptote where that factor equals zero.

Example 4:

Find any vertical asymptotes or holes for $f(x) = \dfrac{x^2 - 3x - 10}{x^2 - x - 6}$.

Example 5:

Find any vertical asymptotes or holes for $f(x) = \dfrac{x^2 + x - 20}{x - 4}$

Finding Horizontal Asymptotes:

of $R(x) = \dfrac{f(x)}{g(x)}$

__Case 1__: deg(f)<deg(g): horizontal asymptote is $y = 0$ (the x-axis).

Example 6: $f(x) = \dfrac{x^2 + 4x}{5x - x^3}$

__Case 2__: deg(f)=deg(g): horizontal asymptote is $y = \dfrac{a}{b}$, where

Example7: $g(x) = \dfrac{3x^2 + 5x}{4x^2 - 1}$

- a is leading coefficient of numerator.
- b is leading coefficient of denominator.

__Case 3__: deg(f)>deg(g): no horizontal asymptote.

Example 8: $R(x) = \dfrac{x^9 + 3x^2}{x^6 + 4x^5 - 5}$

Example 9:

Identify the holes, vertical and horizontal asymptotes.

a. $f(x) = \frac{3x + 1}{x + 3}$

b. $f(x) = \frac{4x}{x^2 - 5x + 4}$

c. $f(x) = \frac{-5}{x^2 - 25}$

Graphing rational functions:

1. Factor numerator and denominator.

2. Find x-intercept(s) by setting numerator equal to zero.
   Note: if a factor "cancels", it results in a hole instead of an x-intercept.

3. Find y-intercept (if any) by substituting $x = 0$ into the original form of the function.
   Note: this is easier if you use the not factored form.

4. Find horizontal asymptote (if any). There can be at most one horizontal asymptote.

5. Find vertical asymptotes (if any) by setting the denominator equal to zero.
   Remember: if a factor in the denominator "cancels", it results in a hole instead of a vertical asymptote.

6. Use the x-intercepts and vertical asymptotes to divide the x-axis into intervals.

7. Choose a test point in each interval to determine if the function is positive or negative there. This will tell you whether the graph approaches the vertical asymptote in an upward or downward direction.

8. Graph! *Except for the breaks at the vertical asymptotes, the graph should be a nice smooth curve with no sharp corners.*