

EE468G NOTES (5C)

Reading assignment:

Contents: Magnetic material and inductance

Magnetic material and boundary condition

Rotation of electron around nucleus form magnetic dipole \vec{M}_k .

The total magnetic dipole per unit volume is defined as the magnetization vector:

$$\text{Magnetization vector: } \vec{M} = \lim_{\Delta v \rightarrow 0} \left(\frac{\sum_k \vec{M}_k}{\Delta v} \right), \quad [\text{A/m}]$$

The magnetic potential produced by the magnetization vector is

$$\vec{A}_m = \frac{\mu_0}{4\pi} \int_V \frac{\nabla' \times \vec{M}}{R} dV' + \frac{\mu_0}{4\pi} \int_S \frac{\vec{M} \times \hat{a}_n}{R} dS'$$

Meanwhile, the magnetic potential produced by the imposed volume current and surface current is given by

$$\vec{A}_i = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J}}{R} dV' + \frac{\mu_0}{4\pi} \int_S \frac{\vec{J}_s}{R} dS'$$

We conclude that the effect of magnetization vector is equivalent to an induced volume and surface current:

$$\begin{aligned} \vec{J}_M &= \nabla \times \vec{M} \\ \vec{J}_{SM} &= \vec{M} \times \hat{a}_n, \end{aligned} \quad \hat{a}_n \text{ is the outward normal direction of S.}$$

Using the induced currents in the Ampere's law, we have

$$\nabla \times \vec{B} = \mu_0 \vec{J}_{\text{total}} = \mu_0 (\vec{J} + \vec{J}_M) = \mu_0 \vec{J} + \mu_0 \nabla \times \vec{M}$$

This means: $\nabla \times (\vec{B} / \mu_0 - \vec{M}) = \vec{J}$

Comparing this equation with Maxwell equation: $\nabla \times \vec{H} = \vec{J}$,
Hence we established that

$$\vec{H} = \vec{B} / \mu_0 - \vec{M}, \quad \text{or} \quad \vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

Simple magnetic material (isotropic), \vec{M} is parallel to \vec{H} ,

$$\vec{M} = \mu_0 \chi_m \vec{H} \Rightarrow \vec{B} = \mu_0 \vec{H} + \mu_0 \chi_m \vec{H} = \mu \vec{H}$$

$$\mu = \mu_0 (1 + \chi_m)$$

Example: A material has $\mu_r = 2000$ and atom density of $2 \times 10^{27} [\text{m}^{-3}]$. If it is put in a uniform magnetizing field with $B_m = 0.15 [\text{T}]$, (1) calculate \vec{M} and the magnetic moments per atom.

Solution: (1) From $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$, we have

$$\vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H} = \frac{\vec{B}}{\mu_0} - \frac{\vec{B}}{\mu_0 \mu_r} = \frac{1}{\mu_0} \left(1 - \frac{1}{\mu_r} \right) \vec{B}$$

M is in the same direction of B , hence

$$M = \frac{1}{4\pi \times 10^{-7}} \left(1 - \frac{1}{2000} \right) \times 0.15 = 0.12 \times 10^6 [\text{A/m}]$$

(2) Moment per atom is: $m = \frac{0.12 \times 10^6}{2 \times 10^{27}} = 0.06 \times 10^{-21} [\text{A-m}]$