9/15/14 MATH 2850

Ch. 14.2

$$\lim_{(x,y)\to(a,b)} f(x,y) = L$$

* $\lim_{(x,y)\to(1,1)} \frac{x^2-y^2}{x-y} = \frac{0}{0} = \lim_{(x,y)\to(1,1)} \frac{\cancel{(x-y)}(x+y)}{\cancel{(x-y)}} = \lim_{(x,y)\to(1,1)} (x+y) = 2$

* $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2} = \frac{0}{0}$

* switch to polar

$x = r\cos\theta \qquad x^2+y^2 = r^2$

$y = r\sin\theta$

$\lim_{r\to 0} \frac{r^2\cos^2\theta - r^2\sin^2\theta}{r^2}$ $\qquad \lim_{r\to 0} \cos^2\theta - \sin^2\theta$

$= \cos^2\theta - \sin^2\theta$ undeterminite : doesn't exist

* $\lim_{(x,y)\to(0,0)} \frac{2x}{x^2+x+y^2} = \frac{0}{0}$

$x = r\cos\theta$

$y = r\sin\theta \qquad x^2+y^2 = r^2$

$= \lim_{r\to 0} \frac{2r\cos\theta}{\cancel{r^2\cos^2\theta + r\cos\theta + r^2\sin^2\theta}\; r^2 + r\cos\theta} = \lim_{r\to 0} \frac{2r\cos\theta}{r(r+\cos\theta)} = \lim_{r\to 0} \frac{2\cos\theta}{r+\cos\theta} = 2$

* $\lim_{x\to 0} \frac{2x}{x^2+x} = \lim_{x\to 0} \frac{2x}{(x+1)x} = \lim_{x\to 0} \frac{2}{x+1} = 2$

* $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2} = \frac{0}{0}$ $\qquad y = mx \qquad \lim_{x\to 0} \frac{x^2mx}{x^4+m^2x^2}$

$\lim_{x\to 0} \frac{x^2mx}{x^2(x^2+m^2)} \qquad \lim_{x\to 0} \frac{mx}{x^2+m^2} = 0$ Don't do this!!

$y = kx^2$

$\lim_{x\to 0} \frac{x^2kx^2}{x^4+k^2x^4} \qquad \lim_{x\to 0} \frac{\cancel{x^4}k}{\cancel{x^4}(1+k^2)} = \frac{k}{1+k^2}$

- <u>Recall</u> A function f as a single variable is differentiable at a point x if there is a constant $f'(x)$ (a number) so that:

$$f(x+\Delta x) = f(x) + f'(x)\Delta x + \varepsilon(x,\Delta x)$$

where $\frac{|\varepsilon(x,\Delta x)|}{|\Delta x|} \to 0$ as $|\Delta x| \to 0$

- <u>Definition</u>: a function, F, of more then one variable is differentiable at a point, P. If there is a constant $f'(p)$ (a thingy) so that:

$$f(p+\Delta p) = f(p) + f'(p)\Delta p + \varepsilon(p,\Delta p)$$

$$\frac{|\varepsilon(p,\Delta p)|}{|\Delta p|} \to 0 \quad \text{as } |\Delta p| \to 0$$

$$\Delta p = \begin{pmatrix} \Delta x \\ \Delta y \\ \vdots \end{pmatrix} \qquad f'(p) = (f_x(p) \;\; f_y(p) \; \cdots)$$

$$(f_x(p) \;\; f_y(p) \cdots)\begin{pmatrix} \Delta x \\ \Delta y \\ \vdots \end{pmatrix}$$

$$= f_x(p)(\Delta x) + f_y(p)\Delta y + \cdots$$