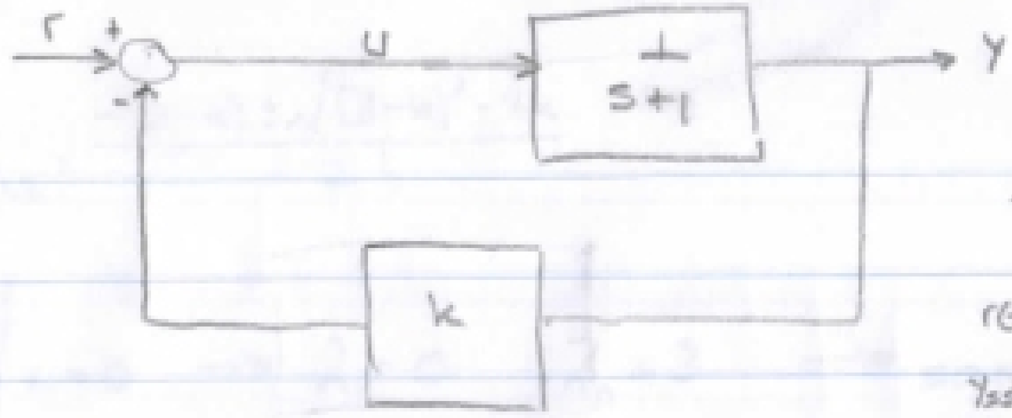
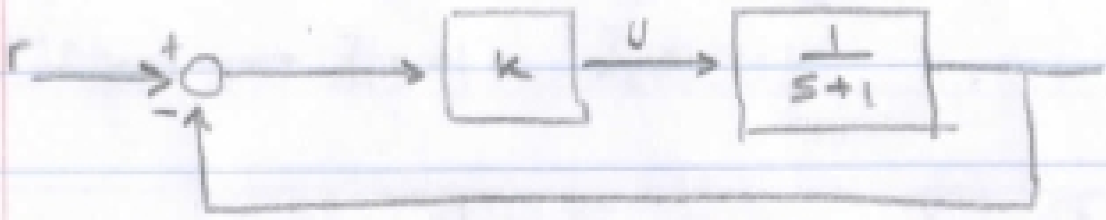


$$TF = \frac{Y(s)}{r(s)} = \frac{\frac{1}{s+1}}{k+\frac{1}{s+1}} = \frac{1}{s+k+1}$$

$r(t)=1 \longrightarrow y_{ss} = ?$

$$y_{ss} = \lim_{s\to 0} \frac{1}{s} s \frac{1}{s+k+1} = \frac{1}{k+1}$$



$$TF = \frac{y(s)}{r(s)} = \frac{\frac{k}{s+1}}{k+\frac{1}{s+1}} = \frac{k}{s+k+1}$$

$r(t)=1 \rightarrow y_{ss} = ?$

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Root Locus

$r=0$

case ① $u=-ky \rightarrow$ output feedback

case ② $u=-k\underline{x} \rightarrow$ state feedback

output feedback

Closed Loop $A_{cl} = A - kBC$

$$\dot{x} = Ax + Bu \rightarrow u=-ky \rightarrow Ax + B(-ky)$$
$$y = Cx + Du \rightarrow y = Cx$$
$$\rightarrow Ax + B-k(Cx) = \underbrace{(A-kBC)}_{A_{cl}}x$$

Eigenvalues of closed loop system

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} -1 \\ 4 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$kB = \begin{bmatrix} -k \\ 4k \end{bmatrix} \rightarrow kBC = \begin{bmatrix} -k \\ 4k \end{bmatrix}\begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} -k & 0 \\ 4k & 0 \end{bmatrix}$$

$$A-kBC = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix} - \begin{bmatrix} -k & 0 \\ 4k & 0 \end{bmatrix} = \begin{bmatrix} k & 1 \\ -4k & -3 \end{bmatrix}$$

$$\lambda I - A_{cl} = \begin{bmatrix} \lambda-k & -1 \\ 4k & \lambda+3 \end{bmatrix}$$

$$|\lambda I - A_{cl}| = \lambda^2 + (3-k)\lambda - 3k + 4k = 0 \longrightarrow \lambda^2 + (3-k)\lambda + k = 0$$

$$\lambda_{1,2} = \frac{-(3-k) \pm \sqrt{(3-k)^2 - 4k}}{2}$$

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if $k=0 \rightarrow \lambda_1 = 0 \quad \lambda_2 = 3$ → same as open loop system!
which is expected b/c we are not controling anything.

if $k=1 \rightarrow \lambda_1 = -1 \quad \lambda_2 = -1$

if $k=2 \rightarrow \lambda_1 = \frac{-1}{2} + j\frac{\sqrt{7}}{2} \quad \lambda_2 = \frac{-1}{2} - j\frac{\sqrt{7}}{2}$
→ osillating, but in stable

if $k=3 \rightarrow \lambda_1 = \frac{j\sqrt{12}}{2} \quad \lambda_2 = -j\frac{\sqrt{12}}{2}$
→ no real part, no decay

if $k=4 \rightarrow \lambda_1 = \frac{1}{2} + j\frac{\sqrt{15}}{2} \quad \lambda_2 = \frac{1}{2} - j\frac{\sqrt{15}}{2}$
→ positive real thus unstable, osillating rise

- This system has 2 poles and one zero
  - by changing k we move the poles

Find Root locus of a cascade of cylindrical tanks

$$A = \begin{bmatrix} \frac{-c}{A} & 0 \\ \frac{c}{A} & -\frac{c}{A} \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{A} \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$q_{in} = -ky \longrightarrow A_{cl} = A - kBC$$

$$kBC = k\begin{bmatrix} \frac{1}{A} \\ 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{k}{A} \\ 0 & 0 \end{bmatrix}$$

$$A_{cl} = A - kBC = \begin{bmatrix} -\frac{c}{A} & 0 \\ \frac{c}{A} & -\frac{c}{A} \end{bmatrix} - \begin{bmatrix} 0 & \frac{k}{A} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -\frac{c}{A} & -\frac{k}{A} \\ \frac{c}{A} & -\frac{c}{A} \end{bmatrix}$$

$$\lambda I - A_{cl} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} -\frac{c}{A} & -\frac{k}{A} \\ \frac{c}{A} & -\frac{c}{A} \end{bmatrix} = \begin{bmatrix} \lambda + \frac{c}{A} & \frac{k}{A} \\ -\frac{c}{A} & \lambda + \frac{c}{A} \end{bmatrix}$$

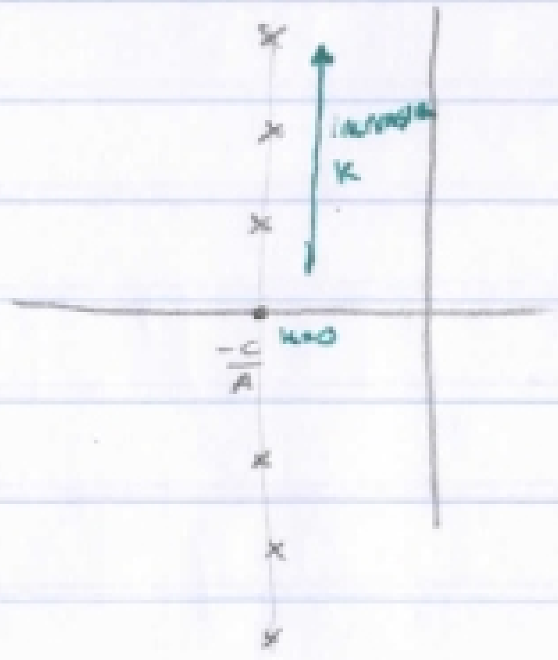
$$\left|\lambda I - A_{cl}\right| = \left(\lambda + \frac{c}{A}\right)\left(\lambda + \frac{c}{A}\right) - \left(\frac{k}{A}\right)\left(-\frac{c}{A}\right)$$

$$\hookrightarrow = \lambda^2 + \frac{2c}{A} + \frac{c^2}{A^2} + \frac{ck}{A^2} = 0$$

$$\lambda_{1,2} = \frac{-c}{A} \pm \sqrt{\frac{c^2}{A^2} - \frac{c^2}{A^2} - \frac{ck}{A^2}} \quad = \lambda_{1,2} = \frac{-c}{A} \pm \frac{\sqrt{-ck}}{A}$$

$$= \lambda_{1,2} = \frac{-c}{A} \pm j\frac{\sqrt{ck}}{A}$$

Root Locus Graph



- on the same line b/c the real part is not a function of time
- damping ratio will never reach 0
- as η goes to 0, ω_n gets larger

Using Simulink, simulate the two tank cascade system on own. (Posted to Blackboard)