COT3100 Summer'2001

Recitation 05/10

Combinatorics.

1. Consider a set of five distinct computer science books, three distinct math books, and two distinct art books.
   a) In how many ways can these books be arranged on a shelf?
   $10!$
   b) In how many ways can these books be arranged on a shelf if all five computer science books are on the left and both art books are on the right?
   $5!\cdot 3!\cdot 2!$
   In how many ways can these books be arranged on a shelf if all books of the same discipline are grouped together?
   $5!\cdot 3!\cdot 2!\cdot 3!$
   c) In how many ways can these books be arranged on a shelf if the two art books are not together?
   There are 9! arrangements where two art books are together. Then there are ($10!-2\cdot 9!$) arrangements where two art books are not together. (9! arrangements for each of two permutations of two art books).
   Ans: $8\cdot 9!$

2. a) How many integers should be selected from the first 10 positive integers (1, 2, …10) to ensure that there exists a pair of these integers with the sum equal 11?
   6

   b) How many integers should be selected from the first 10 positive integers to ensure that there exist at least two pairs of these integers with the sum equal 11?
   7

3. A club has 25 members.
   a) In how many ways the four members can be chosen to serve on an executive committee?
   $C(25, 4) = 25!/(21!\cdot 4!)$
   b) How many ways are there to choose a president, vice president, secretary and treasurer?
   $P(25,4) = 25\cdot 24\cdot 23\cdot 22$

4. How many strings of six lowercase letters from the English alphabet contain

a) the letter *a* (at least one) and any other letters may be used any number of times?
   26^6-25^6

b) the letters *a* and *b* in consecutive positions with *a* preceding *b*, with all the letters distinct?
   $24\cdot 23\cdot 22\cdot 21\cdot 5$

You can represent the result as a two step task. First you select four letters in addition to *a* and *b*. Since each letter can be used only once you have $24\cdot 23\cdot 22\cdot 21$ ways to perform this. The second step is to insert an *ab* tag. You have 5 slots to do this. By the product rule the final answer is the product of these two numbers.

d) the letters *a* and *b*, where *a* is somewhere to the left of *b* in the string, with all letters distinct?
   $24\cdot 23\cdot 22\cdot 21\cdot 15$

Let's consider a two-step task again. First arrange four letters out of 24, and this step can be performed in 24□23□22□21 different ways. The second step is to insert *a* and *b* in 5 slots between four other letters. Since *a* must come to the left of *b*, you perform this task in 5+4+3+2+1 different ways. By the product rule the answer is 24□23□22□21□15

5. Suppose a 5-card poker hand is drawn from the 52-card deck.

a) How many hands contain three cards of one kind and two cards of a second kind?

156□4□6=3744.

Consider any outcome as the three-step procedure. First, there are *P*(13, 2)=13□12=156 ways to select two different kinds out of 13, the kind of the three cards and the kind of two cards. After this you have *C*(4,3)=4 choices for the suites of 3 cards of one kind. And finally, suites for the two cards of the same kind can be selected in *C*(4, 2)=6 different ways. The answer is 156□4□6=3744.

b) How many hands contain all five cards of different kind (any suit)?

1287□4^5= 1,317,888.

Let's first select five different kinds of cards out of 13 available. This task can be done in *C*(13, 5)=13!/(8!□5!)=1287 ways. Then we have four choices for the suit of each card, i. e. there are 4^5 choices to perform the second step. The answer is 1287□4^5= 1,317,888.

c) A flush is a hand when all cards have the same suit. How many five-card flushes are possible?

Ans: 4□13□12□11□10□9/5!=5,148

C(13,5) ways to choose 5 cards from 13 with the same suit. It should be multiplied by 4 different ways to choose a suit.

d) A straight is a hand with five consecutive cards. How many five-card straights are there?

9□4^5.

There are 9 ways to select five consecutive cards. Each card can have four suits, so there are 4^5 choices for suits.

e) How many 5-card straight flushes are there?

36

4 choices for a suit and 9 choices for the 5 consecutive cards from 13.