S. Widnall, J. Peraire
16.07 Dynamics
Fall 2009
Version 2.0

Lecture L7 - Relative Motion using Translating Axes

In the previous lectures we have described particle motion as it would be seen by an observer standing still at a fixed origin. This type of motion is called *absolute* motion. In many situations of practical interest, we find ourselves forced to describe the motion of bodies while we are simultaneously moving with respect to a fixed reference frame. There are many examples where such situations occur. The absolute motion of a passenger inside an aircraft is best described if we first consider the motion of the passenger relative to the aircraft, and then the motion of the aircraft relative to the ground. If we try to track the motion of aircraft in the airspace using satellites, it makes sense to first consider the motion of the aircraft relative to the satellite and then combine this motion with the motion of the satellite relative to the earth's surface. In this lecture we will introduce the ideas of *relative* motion analysis.

Types of observers

For the purpose of studying relative motion, we will consider four different types of observers (or reference frames) depending on their motion with respect to a fixed frame:

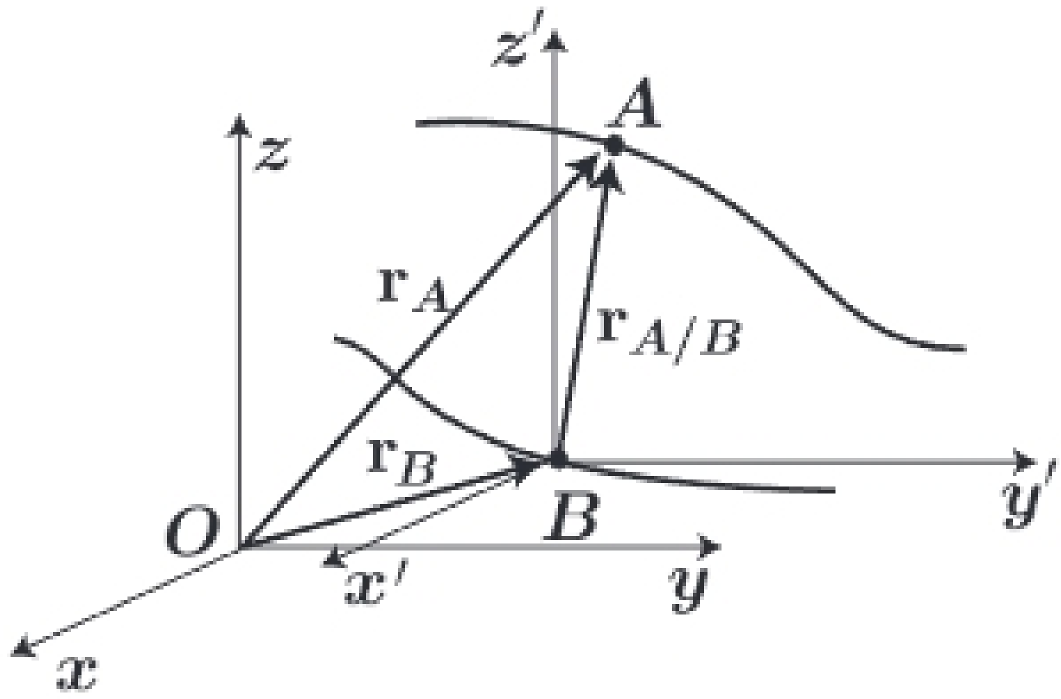
- observers who do not accelerate or rotate, i.e. those who at most have constant velocity.
- observers who accelerate but do not rotate
- observers who rotate but do not translate
- observers who accelerate and rotate

In this lecture we will consider the relative motion involving observers of the first two types, and defer the study of relative motion involving rotating frames to the next lecture.

Relative motion using translating axes

We consider two particles, A and B in curvilinear motions along two different paths. We describe their motion with respect to a fixed reference frame xyz with origin O and with unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$, as before, and call the motion relative to this frame absolute. The position of particle A is given by $\boldsymbol{r}_A(t)$ and the position of particle B is given by $\boldsymbol{r}_B(t)$; both vectors are defined with respect to the fixed reference frame O. In addition, it is useful in many problems to ask "how would B describe the motion of A and how would this description be translate to the fixed inertial coordinate system O?

In order to answer this question, we consider another *translating* reference frame attached to particle B, $x'y'z'$, with unit vectors $\boldsymbol{i}'$, $\boldsymbol{j}'$ and $\boldsymbol{k}'$. Translating means that the angles between the axes xyz and $x'y'z'$ do not change during the motion.



In the figure, we have chosen, for convenience, the axes xyz to be parallel to the axes $x'y'z'$, but it should be clear that one could have non-parallel **translating** axes. (By our ground rules, these axis must not rotate, i.e the angles between them must not change; that possibility will be considered in a subsequent lecture.) The position vector $\boldsymbol{r}_{A/B}$ defines the position of A with respect to point B in the reference frame x'y'z'. The subscript notation"A/B" means "A relative to B". The positions of A and B relative to the absolute frame are given by the vectors $\boldsymbol{r}_A$ and $\boldsymbol{r}_B$, respectively. Thus, we have

$$\boldsymbol{r}_A = \boldsymbol{r}_B + \boldsymbol{r}_{A/B} \ .$$

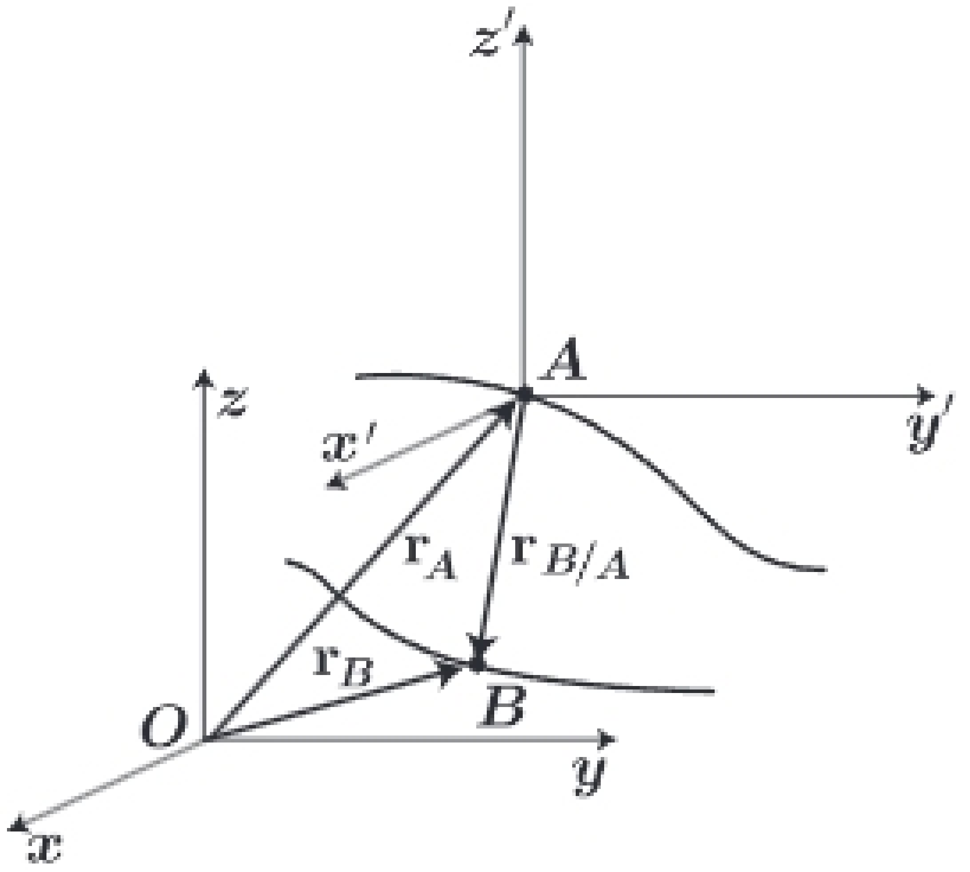
If we derive this expression with respect to time, we obtain

$$\dot{\boldsymbol{r}}_A = \dot{\boldsymbol{r}}_B + \dot{\boldsymbol{r}}_{A/B} \qquad \text{or} \qquad \boldsymbol{v}_A = \boldsymbol{v}_B + \boldsymbol{v}_{A/B} \ ,$$

which relates the absolute velocities $\boldsymbol{v}_A$ and $\boldsymbol{v}_B$ to the relative velocity of A as observed by B. Differentiating again, we obtain an analogous expression for the accelerations,

$$\ddot{\boldsymbol{r}}_A = \ddot{\boldsymbol{r}}_B + \ddot{\boldsymbol{r}}_{A/B} \qquad \text{or} \qquad \boldsymbol{a}_A = \boldsymbol{a}_B + \boldsymbol{a}_{A/B} \ .$$

We will now reverse the roles of A and B , asking "how would A describe the motion of B? We now attach the reference frame $x'y'z'$ to A, and then we can observe B from A. The relative motion of B as seen by A is now denoted $\boldsymbol{r}_{B/A}$, the position of B as seen by A.



The same arguments as before will give us,

$$\boldsymbol{r}_B = \boldsymbol{r}_A + \boldsymbol{r}_{B/A}, \qquad \boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{v}_{B/A}, \qquad \boldsymbol{a}_B = \boldsymbol{a}_A + \boldsymbol{a}_{B/A} \ .$$

Comparing these expressions with those above, we see that

$$\boldsymbol{r}_{B/A} = -\boldsymbol{r}_{A/B}, \qquad \boldsymbol{v}_{B/A} = -\boldsymbol{v}_{A/B}, \qquad \boldsymbol{a}_{B/A} = -\boldsymbol{a}_{A/B} \ ,$$

as expected.

One important observation is that, whenever the moving system, say A, has a constant velocity relative to the fixed system, O, then the acceleration seen by the two observers is the same, i.e., if $\boldsymbol{a}_A = \boldsymbol{0}$, then $\boldsymbol{a}_B = \boldsymbol{a}_{B/A}$. We shall see that this broadens the application of Newton's second law to systems which have a constant absolute velocity.

Example — ***Glider in cross wind***

Consider a glider flying above the edge of a cloud which is aligned North/South (360^o). The glider is flying horizontally, and the cloud is stationary with respect to the ground. At the altitude of the glider flight, there is a wind velocity, $\boldsymbol{v}_w$, of magnitude 52 knots coming from the direction 240^o , as shown in the sketch.