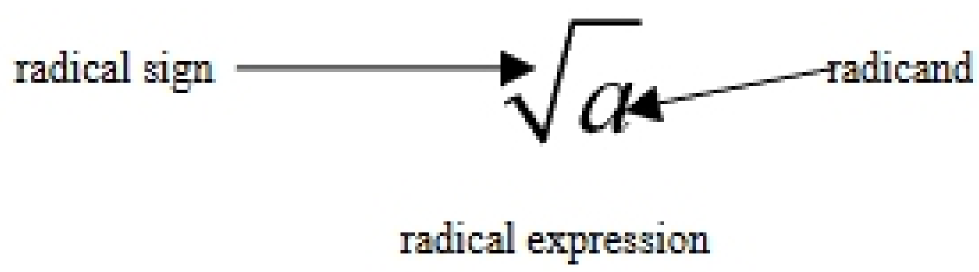
Summer MA 15200 Lesson 4 Section P.3

I Square Roots

If $b^2 = a$, then b is a square root of a.
If a is a nonnegative real number, the nonnegative number b such that $b^2 = a$, denoted by $b = \sqrt{a}$, is the **principal** square root of a.



Ex 1: Evaluate each. If not real, write 'not real'.

$a)\ -\sqrt{81}$

$b)\ \sqrt{\frac{25}{36}}$

$c)\ \sqrt{36+64}$

$d)\ \sqrt{36}+\sqrt{64}$

$e)\ \sqrt{-49}$

Many times students believe that $\sqrt{a^2} = a$. However, the principal square root is always positive. Examine the following.

$\sqrt{8^2} = \sqrt{64} = 8$

$\sqrt{(-8)^2} = \sqrt{64} = 8$, not -8

$-\sqrt{8^2} = -\sqrt{64} = -8$

$\sqrt{-8^2} = \sqrt{-64}$, which is not real

In general:

$$\sqrt{a^2} = |a|$$

Therefore, we will always assume that variables represent positive numbers in order to avoid using absolute value signs.

II Other Types of Roots

$\sqrt[n]{a} = b$ means that $b^n = a$ If n is even, then a and b must be positive. If n is odd, a and b can be any real numbers.

index → $\sqrt[n]{a}$

> If no index is written, the root is assumed to be a square root.

<u>Ex 2</u>: Evaluate each. If not real, write 'not real'.

$a)$ $\sqrt[3]{-125}$

$b)$ $\sqrt[4]{-81}$

$c)$ $\sqrt[6]{64}$

$d)$ $\sqrt[3]{\dfrac{27}{8}}$

$e)$ $\sqrt{0.04}$

$f)$ $\sqrt[5]{-32}$

III The Product and Quotient Rules of Radicals

> If all expressions represent real numbers,
>
> $$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab} \text{ and } \sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$
>
> $$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}} \text{ and } \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad (b \neq 0)$$

Note: These properties are for multiplication and division. Similar statements are not true for addition or subtraction. ($\sqrt[n]{a+b} \neq \sqrt[n]{a} + \sqrt[n]{b}$, for example)

<u>Ex 3</u>: Use the product or quotient rules of radicals (if you can) to write as one radical. Simplify, if possible.

$a)$ $\sqrt{3} \cdot \sqrt{10} =$

$b)$ $\dfrac{\sqrt[3]{54}}{\sqrt[3]{2}} =$

$c)$ $\sqrt{5} \cdot \sqrt[3]{2} =$

IV Simplifying Square Root Radicals

A square root is simplified when its radicand has no factors other than 1 that are perfect squares.

Remember: $\sqrt{a^2} = a$, if a is assumed to be positive. We will assume all variables represent positive values.

Ex 4: Use factoring and the product (and/or quotient)rule to simplify each.

$a)\quad \sqrt{18x^3} =$

$b)\quad \sqrt{81a^7b^5} =$

$c)\quad \sqrt{32x}\cdot\sqrt{2x^5} =$

$d)\quad \dfrac{\sqrt{44m^3n^8}}{\sqrt{11mn^5}} =$

V Addition and Subtraction of Square Roots

Two or more square roots can be combined if they have the same radicand. Such radicals are called **like radicals.** Sometime one or more radical must be simplified in order to combine.

Ex 5: Simplify and combine where possible.

$a)\quad \sqrt{32}+\sqrt{162} =$

$b)\quad 3a\sqrt{3a}-\sqrt{48a^3} =$

$c)\quad 4\sqrt{6a^3}-3a\sqrt{54a} =$