Induction and Recursion.

Explicit definition	Recursive definition
• Factorial $n! = n \cdot (n-1) \dots 2 \cdot 1$	$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n-1)! & \text{if } n > 1 \end{cases}$
• Geometric progression $S_n = 1 + q + q^2 + \dots + q^n$	$S_n = \begin{cases} 1 & \text{if } n = 0 \\ S_{n-1} + q^n & \text{if } n > 0 \end{cases}$
• Power of relation on a set $R^n = R \circ R \circ \dots \circ R$	$R^n = \begin{cases} R & \text{if } n = 1 \\ R^{n-1} \circ R & \text{if } n > 1 \end{cases}$

- The *recursive* definition of a function makes reference to earlier versions of itself.

- The main connection between recursion and induction is that objects are defined by means of a natural sequence.

- Induction is usually the best (possibly the only) way to prove results about recursively defined objects.

How to find a closed form for a recursively defined function?

In general there is no ready to use recipe.

Some simple cases are

- Linear function of integer n: $g_1(n) = an+b$

$$g_1(n+1) = a(n+1)+b = g_1(n) + a$$

$$g_1(n) = \begin{cases} b & \text{if } n = 0 \\ g_1(n-1) + a & \text{if } n > 0 \end{cases}$$

- Quadratic function of integer n: $g_2(n) = an^2+bn+c$

$$g_2(n+1) = a(n+1)^2+b(n+1)+c = g_2(n) + 2an+(a+b)$$

$$g_2(n) = \begin{cases} c & \text{if } n = 0 \\ g_2(n-1) + 2an + (a+b) & \text{if } n > 0 \end{cases}$$