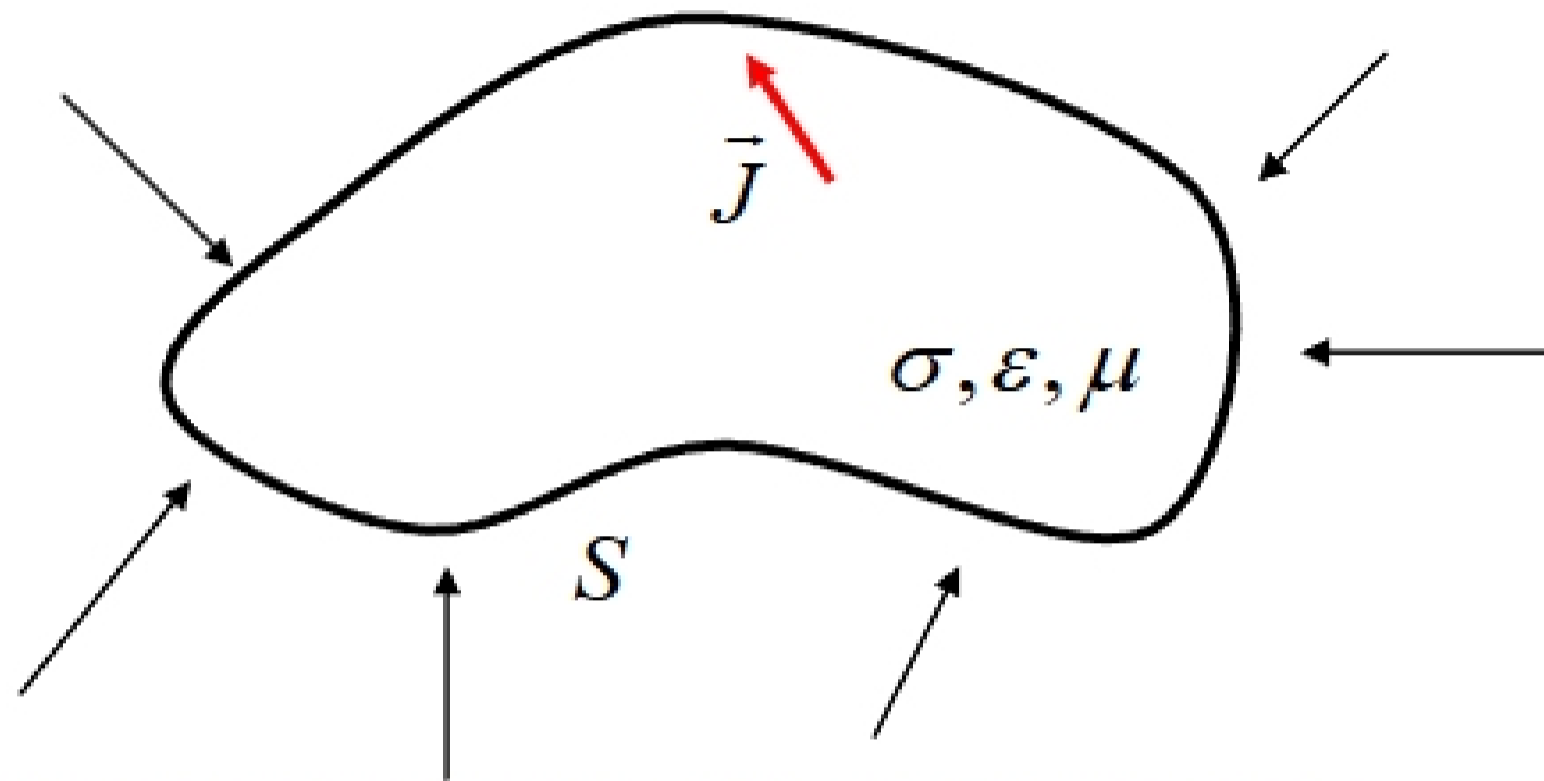


EE468G NOTES (7B)

Contents: Power density and Poynting Vector
Reflection and transmission

Poynting theorem (Power conservation in a closed system): The net power flowing into a closed, source-free system (V) is equal to the sum of the dissipated power and the increasing in energy storage.



This theorem can be expressed in time domain as:

$$-\oint_S \vec{E} \times \vec{H} \cdot d\vec{S} = \int_V \vec{E} \cdot \vec{J} dV + \frac{\partial}{\partial t} \int_V \left[\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \right] dV$$

This equation states that the total power passing through S into V is equal to two parts: one part is the total power dissipation in V, and the other part is the increase in energy storage.

In the above,

$\vec{E} \cdot \vec{J}$ is the dissipated power density [W/m³]

$\frac{1}{2} \epsilon E^2$ is the electric field stored energy density [J/m³]

$\frac{1}{2} \mu H^2$ is the magnetic field stored energy density

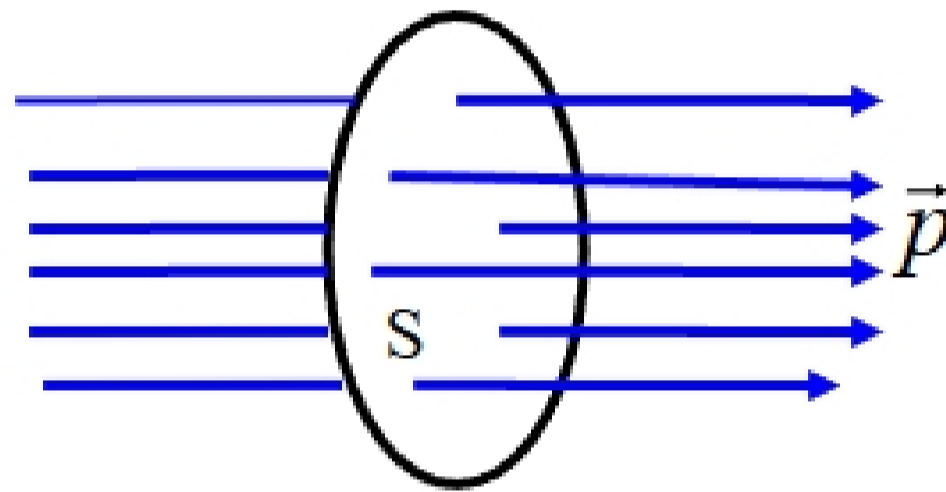
$-\vec{E} \cdot \vec{H}$ is the power flow (in) density.

Complex Poynting vector: $\vec{S} = \vec{E} \times \vec{H}^*$ [W/m²]
 (* stands for complex conjugate).

Time average power flow through a surface:

$$P_{\text{ave}} = \frac{1}{2} \int_S \text{Re}(\vec{E} \times \vec{H}^*) \cdot d\vec{S} \quad [\text{W}]$$

The integrand: $\vec{p}_{\text{ave}} = \frac{1}{2} \text{Re}(\vec{E} \times \vec{H}^*)$ is the power density (W/m²)



For UPW wave in lossless media, the above expression can be simplified. To this end, we note that

$$\vec{H} = \frac{1}{\eta_0} \hat{a}_z \times \vec{E} = \frac{1}{\eta_0} \hat{a}_z \times \vec{E}_0 e^{-j\beta z}$$

Hence

$$\vec{E} \times \vec{H}^* = \vec{E}_0 e^{-j\beta z} \times \left(\frac{1}{\eta_0} \hat{a}_z \times \vec{E}_0^* e^{j\beta z} \right) = \frac{1}{\eta_0} |\vec{E}_0|^2 \hat{a}_z$$

If the wave propagates in k-direction, we will have

$$\vec{E} \times \vec{H}^* = \frac{1}{\eta_0} |\vec{E}_0|^2 \hat{a}_k, \quad P_{\text{ave}} = \frac{1}{2\eta_0} |\vec{E}_0|^2 \int_S \hat{a}_k \cdot d\vec{S}$$

Here, \hat{a}_k is the direction of wave propagation. It is clear that if the power density is in the normal direction of S, then the integral is S, the area of the surface.