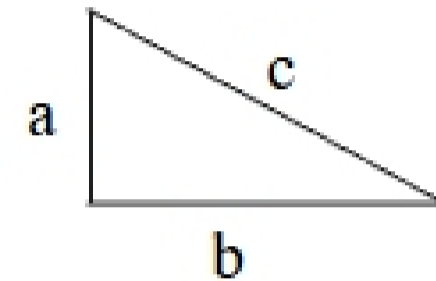


## Math and Physics Refresher

This course assumes that you have studied Newtonian mechanics in a previous calculus-based physics course (i.e. PHY2060) and at least have co-registered in a vector calculus course (Calc 3). Listed below are some of the concepts in basic math, calculus, and physics that you are expected to know *or to acquire* during this course. This is not a complete summary of introductory math and physics. It is only meant to be a refresher of some of the concepts used in this course. Please report any inaccuracies to the professor.

### Geometry



1. Pythagorean Theorem:  
The square of the hypotenuse of a right triangle is the sum of the squares of the two legs:  $c^2 = a^2 + b^2$
2. Circumference of a circle:  $C = 2\pi R$
3. Volume of a sphere:  $V = \frac{4}{3}\pi R^3$
4. Surface area of sphere:  $S = 4\pi R^2$

### Calculus

1. Differentiation:

You are expected to be able to take simple derivatives:

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} e^x = e^x$$

2. Product Rule

$$\frac{d}{dx} [f(x)g(x)] = f(x)\frac{dg}{dx} + g(x)\frac{df}{dx}$$

Example:  $\frac{d}{dx} (x \sin x) = x \cos x + \sin x$

## 3. Chain Rule

$$\frac{d}{dx} f(g(x)) = \frac{df}{dg} \frac{dg}{dx}$$

Examples:

$$\frac{d}{dx} \sin 2x = 2 \cos 2x$$

$$\frac{d}{dx} \exp(-x^2) = -2x \exp(-x^2)$$

## 4. Integration

You are expected to be able to perform simple integrals:

$$\int x^n dx = \frac{1}{n+1} x^{n+1}$$

$$\int dx \cos x = \sin x$$

$$\int dx \sin x = -\cos x$$

$$\int dx e^x = e^x$$

A purist would note that a constant should be added to these indefinite integrals.

## 5. Change of Variables

To use integration tables correctly, you must be able to change variables. For example:

$$I = \int_0^L \sin^2\left(\frac{n\pi x}{L}\right)$$

$$\text{Let } u = \frac{n\pi x}{L} \Rightarrow dx = \frac{L}{n\pi} du$$

$$I = \frac{L}{n\pi} \int_0^{n\pi} du \sin^2 u$$

$$\text{Then use } \int dx \sin^2 x = \frac{x}{2} - \frac{1}{4} \sin 2x$$

$$I = \frac{L}{n\pi} \frac{n\pi}{2} = \frac{L}{2}$$

## 6. Integration By Parts

$$\int u dv = uv - \int v du$$

## Approximations

For small  $x$ , the following expansions are useful:

1.  $\frac{1}{1-x} \approx 1 + x + x^2 + \dots$
2. Binomial Expansion:  $(1+x)^n \approx 1 + nx + \dots$
3. Taylor Expansion:  $f(x) \approx f(0) + x \left. \frac{df}{dx} \right|_{x=0} + \frac{x^2}{2!} \left. \frac{d^2 f}{dx^2} \right|_{x=0} + \dots$

## Differential Equations

We will study the solutions to several differential equations when we study circuits in this class. Although you are not required to have taken a course in differential equations, we will learn how to solve the simplest ones:

1. The exponential function is the only function whose derivative is the function itself:

$$\frac{df}{dx} = \alpha f(x)$$

$f(x) = Ce^{\alpha x}$  is the general solution, where  $C$  and  $\alpha$  are constants

2. Two derivatives of the trigonometric functions give you back the same function with a sign change:

$$\frac{d^2 f}{dx^2} = -k^2 f(x)$$

$f(x) = A \sin kx + B \cos kx$  is the general solution, where  $A$ ,  $B$ , and  $k$  are constants

Using complex exponentials (see next section), you can also represent this solution as:

$$f(x) = A' e^{ikx} + B' e^{-ikx}$$