# Mathematical Functions

In mathematics, a function is an equation where you "plug in" a value, and get an "answer" so to speak. In particular, whenever you plug in a particular value, you must get a SINGLE answer. (You should also get the same answer always.) Functions graphed on the x-y plane have to pass the vertical line test.

Now, in discrete mathematics, we will be using functions a bit differently & we will also coin a new term "relation". In particular, a function is a specific type of relation.

In standard high school mathematics, we typically deal with functions of one variable. We always graph a function of the form y=f(x), where the left hand side is entirely dependent on x. Depending on what the function f(x) is, there is always a set of values that are VALID to "plug" in to the equation. This set is the domain. Similarly, the "answer" you get out of the function will always lie in a particular set. This set is the range.

The problem with using standard functions for discrete mathematics is that many are defined for all real numbers. Namely, it would be nice if we could list every value in the domain of some function. But, we CAN NOT list out each real number. (We can list out each integer however...)

The basis of functions and relations in discrete mathematics is the idea that values of a domain and range should be subsets of a set that can be listed, such as the integers, color, etc.

As we go through different things, I will make analogies to mathematical functions, so you can see the similarities between these and the functions and relations for discrete mathematics.

# Relations

A relation is something that relates one set of values to another set of values. Sometimes the relationship that is specified between sets is meaningful, other times it is not.

In general, a relations are defined in the following manner:

A relation R defined over sets A and B is a subset of A x B. Thus, we have R $\subseteq$ A x B. This is known as a binary relation, because it relates elements between two sets.

Consider this example:

Let A = {Orange Juice, Cranberry Juice, Coke} and
B = {Rum, Vodka, Peach Schnapps}

If you had some modicum of taste, we could define a relation Cocktails as follows:

Cocktails = { (Orange Juice, Vodka),(Cranberry Juice, Vodka),
(Coke, Rum), (Orange Juice, Peach Schnapps) }

Of course, if you do not have any standards, we could have up to 9 pairs listed in our relation for Cocktails.

Graphically, we could use a directed graph to represent this information as follows:

**Of course, you can see there are some restrictions with only being able to define binary relations. For example, even if we extended our sets A and B from the previous example to provide for fully stocked bar, we STILL could not define a relation that would include a Long Island Ice Tea. (For any one not familiar with this drink, it contains 4 elements from an extended version of set B.)**

**Thus, we should define relations between more than two items. In general, we can define an n-ary relation as follows:**

**An n-ary relation R over sets $A_1$, $A_2$, $A_3$, ... $A_n$ is a subset of the cartesian product $A_1$ x $A_2$ x $A_3$ ... x $A_n$. The degree of this relation R is n.**

**Now, we could define a relation on A x A x B x B x B x B that would include a Long Island Ice tea as an element of it.**

**Of course, it is probably more typical that an n-ary relation be comprised of several different sets, but there is no rule against defining a relation using the same set repeatedly, as we have done above.**

**Also, we can denote an n-ary relation using a table as follows:**

| Mixer 1 | Mixer 2 | Liquor 1 | Liquor 2 | Liquor 3 | Liquor 4 |
|---|---|---|---|---|---|
| Coke | Lemon Juice | Vodka | Tequila | Rum | Gin |
| ... | ... | ... | ... | ... | ... |