

Z, Z and Z-R Relationships

$$P_r = \left(\frac{G^2 \lambda^2 P_t \theta \phi c \tau}{1024 \ln(2) \pi^2} \right) \frac{\eta}{R^2} = C \frac{\eta}{R^2}$$

$$\eta = \frac{\pi^5 |K|^2}{\lambda^4} z$$

$$z = \sum_{\text{Unit Volume}} D^6$$

Radar equation for distributed targets

Radar reflectivity

Radar reflectivity factor
(mm^6/m^3)

Combining these equations, we can write: $P_r = c_2 \frac{z}{R^2}$, and rearrange to get:

$$z = c_3 P_r R^2$$

c_3 is the so-called *radar constant*. It has units $\text{mm}^6/\text{m}^3 \text{ mW}^{-1} \text{ km}^{-2}$. The radar reflectivity factor z has a tremendous dynamic range so it is convenient to express it on a decibel scale with a reference $z = 1 \text{ mm}^6/\text{m}^3$

$$Z = 10 \log_{10} \left(\frac{z (\text{mm}^6 / \text{m}^3)}{1 (\text{mm}^6 / \text{m}^3)} \right) \text{ with units dBZ}$$

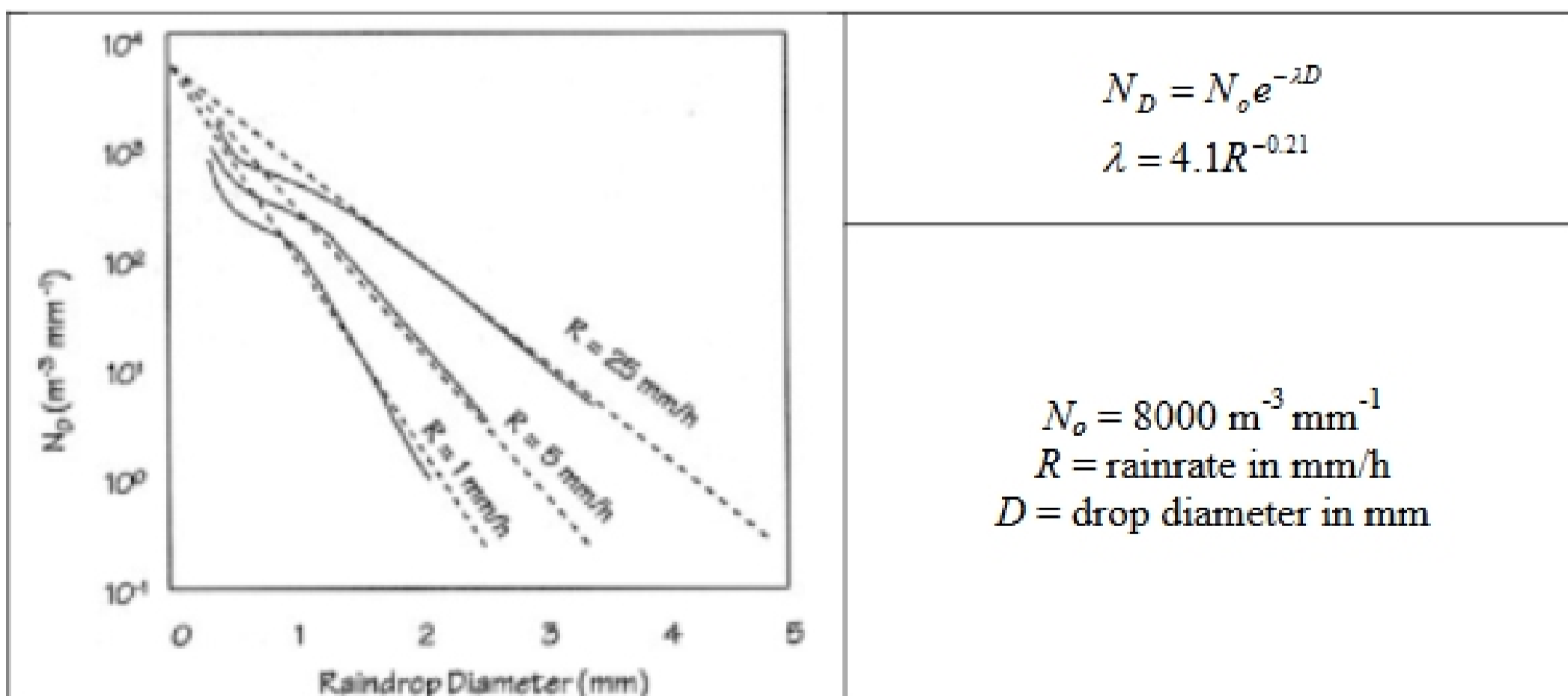
Using this, we can write

$$z = c_3 P_r R^2$$

$$Z = C_3 + P_r + 20 \log_{10}(R)$$

where Z is measured in dBZ, $C_3 = 10 \log_{10}(c_3)$, P_r is measured in dBm, and R is in km.

Marshall-Palmer DSD.



Given a DSD one can compute a Z-R relationship. In practice, empirical relationships are used:

$$Z = AR^b$$

$$Z = 200R^{1.6} \text{ Marshall - Palmer or MP relationship}$$

There are many relationship in use. The table below is from the NOAA ROC:

Table 1. Z-R RECOMMENDATIONS		
RELATIONSHIP	Optimum for:	Also recommended for:
Marshall-Palmer ($z=200R^{1.6}$)	General stratiform precipitation	
East-Cool Stratiform ($z=130R^{2.0}$)	Winter stratiform precipitation - east of continental divide	Orographic rain - East
West-Cool Stratiform ($z=75R^{2.0}$)	Winter stratiform precipitation - west of continental divide	Orographic rain - West
WSR-88D Convective ($z=300R^{1.4}$)	Summer deep convection	Other non-tropical convection
Rosenfeld Tropical ($z=250R^{1.2}$)	Tropical convective systems	

Sample Z-R relationships measured in Iowa (note the X-Y axis are switched between the graphs).

