

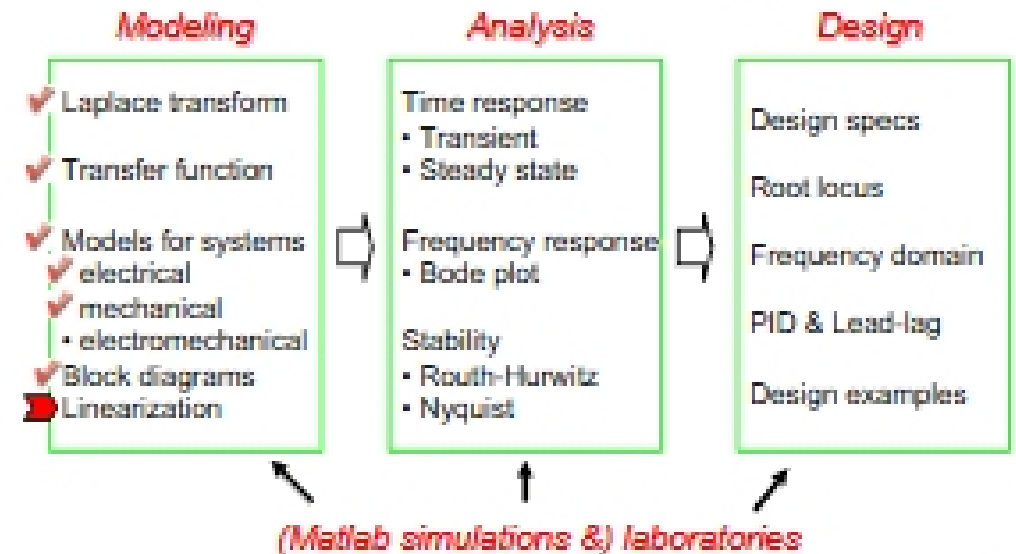
ME451: Control Systems

Lecture 7 Linearization, time delays

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1

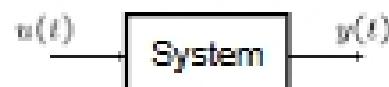
Course roadmap



2

What is a linear system?

- A system having **Principle of Superposition**



$$\left. \begin{array}{l} u_1(t) \rightarrow y_1(t) \\ u_2(t) \rightarrow y_2(t) \end{array} \right\} \Rightarrow \alpha_1 u_1(t) + \alpha_2 u_2(t) \rightarrow \alpha_1 y_1(t) + \alpha_2 y_2(t) \\ \forall \alpha_1, \alpha_2 \in \mathbb{R}$$

A **nonlinear system** does not satisfy the principle of superposition.

3

Linear systems

- Easier to understand and obtain solutions
- Linear ordinary differential equations (ODEs),
 - Homogeneous solution and particular solution
 - Transient solution and steady state solution
 - Solution caused by initial values, and forced solution
- Add many simple solutions to get more complex ones (use superposition!)
- Easy to check the **Stability** of stationary states (Laplace Transform)

4

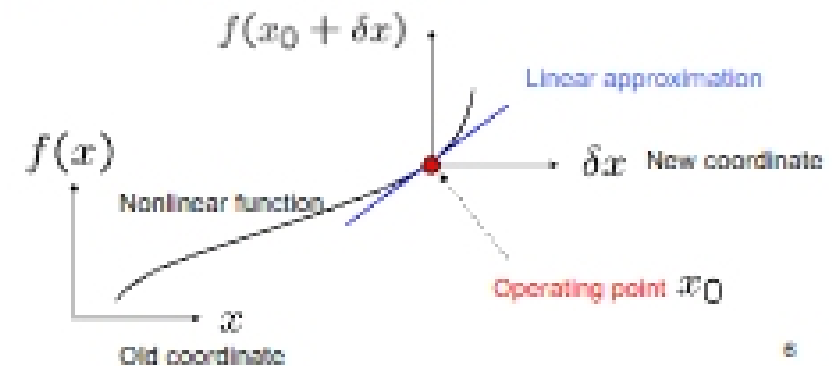
Why linearization?

- Real systems are inherently nonlinear. (Linear systems do not exist!) Ex. $f(t)=Kx(t)$, $v(t)=Ri(t)$
- TF models are only for linear time-invariant (LTI) systems.
- Many control analysis/design techniques are available for linear systems.
- Nonlinear systems are difficult to deal with mathematically.
- Often we linearize nonlinear systems before analysis and design. How?

5

How to linearize it?

- Nonlinearity can be approximated by a linear function for small deviations δx around an operating point x_0
- Use a Taylor series expansion



6

Linearization

- Nonlinear system: $\dot{x} = f(x, u)$
- Let u_0 be a nominal input and let the resultant state be x_0
- Perturbation: $u(\cdot) = u_0(\cdot) + \delta u(\cdot)$
- Resultant perturb: $x(\cdot) = x_0(\cdot) + \delta x(\cdot)$
- Taylor series expansion:

$$f(x, u) = f(x_0, u_0) + \left. \frac{\partial f(x, u)}{\partial x} \right|_{\substack{x=x_0 \\ u=u_0}} \delta x + \left. \frac{\partial f(x, u)}{\partial u} \right|_{\substack{x=x_0 \\ u=u_0}} \delta u + \underbrace{\text{H.O.T.}}_{\approx 0}$$

7

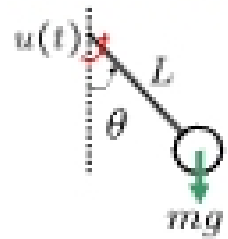
Linearization (cont.)

notice that $\dot{x}_0 = f(x_0, u_0)$; hence

8

Linearization of a pendulum model

- **Motion of the pendulum**



$$mL^2\ddot{\theta}(t) + mgL \sin \theta(t) = u(t)$$

$$\ddot{\theta}(t) + \underbrace{\frac{g \sin \theta(t)}{L} - \frac{u(t)}{mL^2}}_{f(\theta, u)} = 0$$

- **Linearize it at** $\theta_0 = \pi$
- **Find u_0** $\bar{\pi} + \frac{g \sin \pi}{L} - \frac{u_0}{mL^2} = 0 \rightarrow u_0 = 0$
- **New coordinates:**

$$\theta = \theta_0 + \delta\theta = \pi + \delta\theta$$

$$u = u_0 + \delta u = 0 + \delta u$$

9

Linearization of a pendulum model (cont')

- **Taylor series expansion of**

$$\left. \frac{\partial f(\theta, u)}{\partial \theta} \right|_{\substack{\theta=\pi \\ u=0}} = \left. \frac{g \cos \theta}{L} \right|_{\theta=\pi} = -\frac{g}{L}$$

$$\left. \frac{\partial f(\theta, u)}{\partial u} \right|_{\substack{\theta=\pi \\ u=0}} = -\frac{1}{mL^2}$$

$$\delta\ddot{\theta} + \left. \frac{\partial f(\theta, u)}{\partial \theta} \right|_{\theta=\pi} \delta\theta + \left. \frac{\partial f(\theta, u)}{\partial u} \right|_{\theta=\pi} \delta u = 0$$

10