2.004 Dynamics and Control II
Spring 2008

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING

2.004 *Dynamics and Control II*
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Lecture 30[1]

Reading:

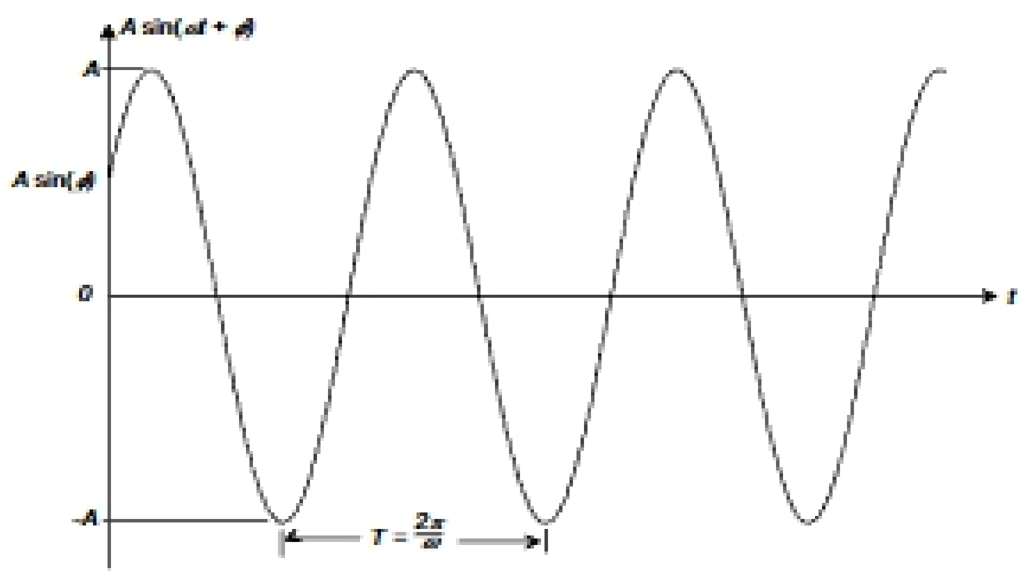
- Nise: 10.1

1 Sinusoidal Frequency Response

1.1 Definitions

Consider a sinusoidal waveform

$$f(t) = A\sin(\omega t + \phi)$$



where

A is the amplitude (in appropriate units)

ω is the angular frequency (rad/s)

ϕ is the phase (rad)

In addition we can define

T the period $T = 2\pi/\omega$ (s)

f the frequency, $(f = 1/T = \omega/2\pi)$ (Hz)

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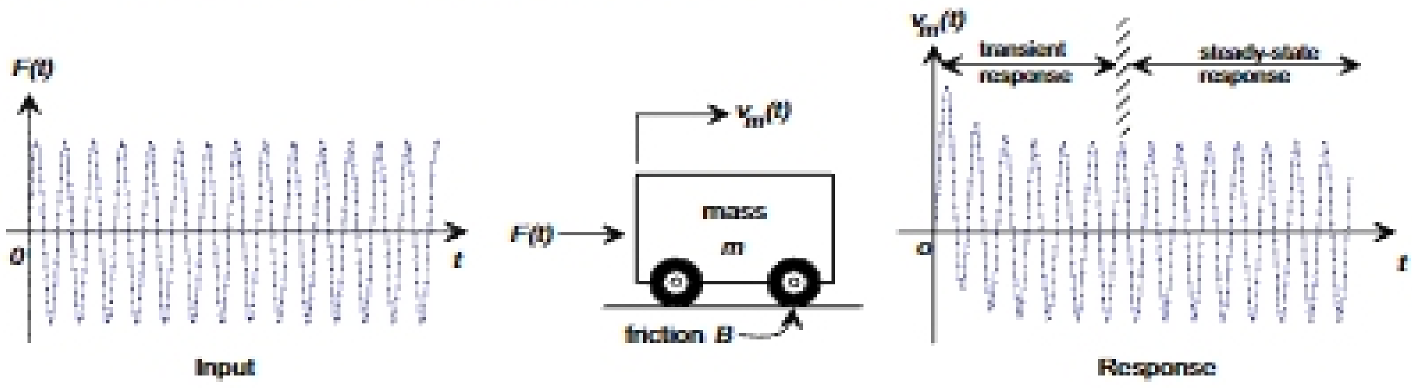
The Euler Formulas: We will frequently need the Euler formulas

$$\begin{aligned} e^{j\omega t} &= \cos(\omega t) + j\sin(\omega t) \\ e^{-j\omega t} &= \cos(\omega t) - j\sin(\omega t) \end{aligned}$$

or conversely

$$\begin{aligned} \cos(\omega t) &= \frac{1}{2}\left(e^{j\omega t} + e^{-j\omega t}\right) \\ \sin(\omega t) &= \frac{1}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right) \end{aligned}$$

1.2 The Steady-State Sinusoidal Response



Assume a system, such as shown above, is excited by a sinusoidal input. The total response will have two components a) a transient component, and a steady-state component

$$y(t) = y_h(t) + y_p(t).$$

We define the steady-state component as the particular solution $y_p(t)$. Let the system differential equation be

$$a_n\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} + \ldots + a_1\frac{dy}{dt} + a_0 y = b_m\frac{d^m u}{dt^m} + b_{m-1}\frac{d^{m-1}u}{dt^{m-1}} + \ldots + b_1\frac{du}{dt} + b_0 u$$

with a complex exponential input

$$u(t) = e^{j\omega t}.$$

Assume a particular solution $y_p(t)$ to be of the same form as the input, that is

$$y_p(t) = Ae^{j\omega t}$$

and since

$$\frac{d^k y_p}{dt^k} = A(j\omega)^k e^{j\omega t}$$

substitution into the differential equation gives:

$$\begin{aligned} &\left(a_n(j\omega)^n + a_{n-1}(j\omega)^{n-1} + \ldots + a_1(j\omega) + a_0\right) Ae^{j\omega t} \\ &\qquad = \left(b_m(j\omega)^m + b_{m-1}(j\omega)^{m-1} + \ldots + (b_1 j\omega) + b_0\right) e^{j\omega t} \end{aligned}$$