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$$y(0) = 1 \quad y'(0) = 2$$

$$y'' - 4y' + 3y = g(t)$$

$$s^2 \mathcal{L}y - sy(0) - y'(0) - 4(s\mathcal{L}y - y(0)) + 3\mathcal{L}y = \mathcal{L}g(t)$$

$$(s^2 - 4s + 3) \mathcal{L}y - s - 2 + 4 = \mathcal{L}y g(t)$$

$$\mathcal{L}y = \frac{1}{s} g(t) \left(\frac{1}{s^2 - 4s + 3} \right) + \frac{s}{(s^2 - 4s + 3)} + \frac{2}{s^2 - 4s + 3}$$

$$g(t) \left(\frac{1}{(s-3)(s-1)} \right) + \frac{s}{(s-3)(s-1)} + \frac{2}{(s-3)(s-1)}$$

$$g(t) \left(\frac{1}{2} e^{3t} - \frac{1}{2} e^t \right) - (e^{3t} - e^t) +$$

$$\frac{s}{s^2 - 4s + 3} = \frac{as - a}{s^2 - 4s + 3} + \frac{bs - b}{s^2 - 4s + 3} = \frac{s}{s^2 - 4s + 3}$$

$$a + b = 1$$

$$-a - 3b = 0$$

$$b = -1/2 \quad a = 3/2$$

$$3/2 \left(\frac{1}{s-3} \right) - 1/2 \left(\frac{1}{s-1} \right)$$

$$g(t) \left(\frac{1}{2} e^{3t} - \frac{1}{2} e^t \right) - (e^{3t} - e^t) + \frac{3}{2} e^{3t} - \frac{1}{2} e^t$$

$$\frac{3}{2} e^{3t} - \frac{1}{2} e^t - e^{3t} + e^t + \int_0^t \left(\frac{1}{2} e^{3(t-\alpha)} - \frac{1}{2} e^{(t-\alpha)} \right) g(\alpha) d\alpha$$

$$as - 3a + bs - b$$

$$-3a - b = 1$$

$$-1 - 3a = b$$

$$as - 3as = 1$$

$$a = 1/2$$

$$b = -1/2$$

$$y'' - 2y' + 2y = g(t) \quad y(0), y'(0) = 0$$

$$(s^2 - 2s + 2) \mathcal{L}y = \mathcal{L}g(t)$$

$$\mathcal{L}y = \mathcal{L}g(t) \left(\frac{1}{(s-1)^2 + 1} \right) \int_0^t g(\alpha) e^{-(t-\alpha)} \sin(t-\alpha) d\alpha$$

$$e^{2t} \downarrow \sin t$$

HW. do Variation of Parameters or UDC

$$g(t) = 1 \quad \text{and plug it in}$$

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$$\text{f.f. if } \mathcal{L}(\sin t) = \frac{1}{s^2 + 1}$$

$$\int_0^t \sin(t-\alpha) \sin \alpha d\alpha = \int_0^t (\sin t \cos \alpha - \cos t \sin \alpha) \sin \alpha d\alpha$$

$$\sin t \int_0^t \cos \alpha \sin \alpha d\alpha - \cos t \int_0^t \sin^2 \alpha d\alpha$$

$$\cos(2t) = 1 - \sin^2 t$$

$$\sin t \left(\frac{\sin^2 \alpha}{2} \right) \Big|_0^t - \cos t \int_0^t \sin^2 \alpha d\alpha$$

$$\cos \Big|_0^t \frac{1}{2} \cos(2\alpha)$$

$$\cos t \left(\frac{1}{2} t - \frac{\sin(2t)}{4} \right)$$

$$\frac{\sin^3 t}{2} - \frac{1}{2} \cos t + \frac{\cos t \sin 2t}{4}$$

$$\cos t \left(\frac{1}{2} t - \frac{\sin 2t}{4} \right)$$

$$\frac{\sin^3 t}{2} - \frac{1}{2} \cos t + \frac{\cos t (2 \cos t \sin t)}{4}$$

$$\frac{1}{2} t + \frac{1}{4} \sin 2t \Big|_0^t$$

$$\frac{\sin^3 t}{2} - \frac{1}{2} \cos t + \frac{\cos^2 t \sin t}{2}$$

$$\frac{1}{2} + \frac{1}{4} \sin 2t$$

$$-\frac{1}{2} (\sin t - t \cos t)$$