

56:166 Production Systems

Material Review 4

Scheduling



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Problem 1

For the following matrix of sequence dependent setup times in minutes:

$$[t_{ij}] = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} - & 9 & 4 & 7 & 2 & 10 \\ 11 & - & 9 & 5 & 2 & 6 \\ 5 & 14 & - & 17 & 4 & 9 \\ 4 & 4 & 2 & 7 & - & 5 & 9 \\ 8 & 6 & 4 & 6 & - & 3 \\ 19 & 5 & 4 & 11 & 8 & - \end{bmatrix} \end{matrix}$$



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- (a) Formulate a scheduling model minimizing the total changeover time.
- (b) Find the optimal schedule for manufacturing the six batches of parts
- (c) What is the makespan of the schedule, given the processing time of each batch is 4 minutes.



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The Model

$$\begin{aligned} \min \quad & \sum_{i=1}^n \sum_{j=i}^n c_{ij} x_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^n x_{ij} = 1 \quad i=1, \dots, n \\ & \sum_{j=1}^n x_{ij} = 1 \quad i=1, \dots, n \\ & x_i - x_j + nx_{ij} \leq n-1 \quad i=2, \dots, n; j=2, \dots, n; i \neq j \\ & x_{ij} = 0, 1 \quad i, j=1, \dots, n \\ & x_{ij} \geq 0 \quad i=1, \dots, n \end{aligned}$$



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Solution

a)

$$\begin{aligned} \text{Min } & 9X_{12} + 4X_{13} + 7X_{14} + 2X_{15} + 10X_{16} + \\ & 11X_{21} + 9X_{23} + 5X_{24} + 2X_{25} + 6X_{26} + \\ & 5X_{31} + 14X_{32} + 17X_{34} + 4X_{35} + 9X_{36} + \\ & 4X_{41} + 2X_{42} + 7X_{43} + 5X_{45} + 9X_{46} + \\ & 8X_{51} + 6X_{52} + 4X_{53} + 6X_{54} + 3X_{56} + \\ & 19X_{61} + 5X_{62} + 4X_{63} + 11X_{64} + 8X_{65} \end{aligned}$$



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$$\begin{aligned} X_{12} + X_{13} + X_{14} + X_{15} + X_{16} &= 1 \\ X_{21} + X_{23} + X_{24} + X_{25} + X_{26} &= 1 \\ X_{31} + X_{32} + X_{34} + X_{35} + X_{36} &= 1 \\ X_{41} + X_{42} + X_{43} + X_{45} + X_{46} &= 1 \\ X_{51} + X_{52} + X_{53} + X_{54} + X_{56} &= 1 \\ X_{61} + X_{62} + X_{63} + X_{64} + X_{65} &= 1 \end{aligned}$$

$$\begin{aligned} X_{21} + X_{31} + X_{41} + X_{51} + X_{61} &= 1 \\ X_{12} + X_{32} + X_{42} + X_{52} + X_{62} &= 1 \\ X_{13} + X_{23} + X_{43} + X_{53} + X_{63} &= 1 \\ X_{14} + X_{24} + X_{34} + X_{54} + X_{64} &= 1 \\ X_{15} + X_{25} + X_{35} + X_{45} + X_{65} &= 1 \\ X_{16} + X_{26} + X_{36} + X_{46} + X_{56} &= 1 \end{aligned}$$



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$6X23+U2-U3 \leq 5$   
 $6X24+U2-U4 \leq 5$   
 $6X25+U2-U5 \leq 5$   
 $6X26+U2-U6 \leq 5$   
 $6X32+U3-U2 \leq 5$   
 $6X34+U3-U4 \leq 5$   
 $6X35+U3-U5 \leq 5$   
 $6X36+U3-U6 \leq 5$   
 $6X42+U4-U2 \leq 5$   
 $6X43+U4-U3 \leq 5$   
 $6X45+U4-U5 \leq 5$   
 $6X46+U4-U6 \leq 5$



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$6X52+U5-U2 \leq 5$   
 $6X53+U5-U3 \leq 5$   
 $6X54+U5-U4 \leq 5$   
 $6X56+U5-U6 \leq 5$   
 $6X62+U6-U2 \leq 5$   
 $6X63+U6-U3 \leq 5$   
 $6X64+U6-U4 \leq 5$   
 $6X65+U6-U5 \leq 5$   
 End  
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**b) Solve the mathematical model in LINDO**

OBJECTIVE FUNCTION VALUE

1) 23.00000

VARIABLE	VALUE	REDUCED COST
X12	0.000000	9.000000
X13	0.000000	4.000000
X14	1.000000	7.000000
X15	0.000000	2.000000
X16	0.000000	10.000000
X21	0.000000	11.000000
X23	0.000000	9.000000
X24	0.000000	5.000000
X25	1.000000	2.000000
X26	0.000000	6.000000



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X31	1.000000	5.000000
X32	0.000000	14.000000
X34	0.000000	17.000000
X35	0.000000	4.000000
X36	0.000000	9.000000
X41	0.000000	4.000000
X42	1.000000	2.000000
X43	0.000000	7.000000
X45	0.000000	5.000000
X46	0.000000	9.000000
X51	0.000000	8.000000
X52	0.000000	6.000000
X53	0.000000	4.000000
X54	0.000000	6.000000
X56	1.000000	3.000000



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X61	0.000000	19.000000
X62	0.000000	5.000000
X63	1.000000	4.000000
X64	0.000000	11.000000
X65	0.000000	8.000000
U2	1.000000	0.000000
U3	5.000000	0.000000
U4	0.000000	0.000000
U5	2.000000	0.000000
U6	4.000000	0.000000

$X14 = X25 = X31 = X42 = X56 = X63 = 1$   
 Thus, the machining sequence of the six batches is:  
 1-4-2-5-6-3-1



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c) The schedule is 1-4-2-5-6-3  
 the corresponding changeover time is  
 $t = [7, 2, 2, 3, 4]$

Makespan =  $7 + 2 + 2 + 3 + 4 + 4 \cdot 6 = 42$

$$[t_{ij}] = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} - & 9 & 4 & \textcircled{7} & 2 & 10 \\ 11 & - & 9 & 5 & \textcircled{2} & 6 \\ \textcircled{5} & 14 & - & 17 & 4 & 9 \\ 4 & 4 & \textcircled{2} & 7 & - & 5 & 9 \\ 8 & 6 & 4 & 6 & - & \textcircled{3} \\ 19 & 5 & \textcircled{4} & 11 & 8 & - \end{bmatrix} \end{matrix}$$


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**Problem 2**

Solve optimally (min Fmax) the following job shop scheduling problem with 10 parts.

Part	Processing Order and Time			
	First Machine		Second Machine	
1	M1	8	M2	10
2	M1	8	M2	8
3	M2	10	M1	11
4	M1	7	-	-
5	M2	10	-	-
6	M1	4	M2	8
7	M1	10	-	-
8	M1	4	M2	10
9	M2	9	-	-
10	M2	10	M1	8



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The transportation time is 1 unit for each part transfer.

- (a) How many operations are scheduled in this problem?
- (b) How many precedence constraints are in this problem?
- (c) Draw Gantt chart of the optimal schedule
- (d) What is the schedule makespan?



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**Solution**

a) 16 operations

b) 6 precedence constraints

Part	Processing Order and Time			
	First Machine		Second Machine	
1	M1	8	M2	10
2	M1	8	M2	8
3	M2	10	M1	11
4	M1	7	-	-
5	M2	10	-	-
6	M1	4	M2	8
7	M1	10	-	-
8	M1	4	M2	10
9	M2	9	-	-
10	M2	10	M1	8



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c)

To determine an optimal schedule, separate the parts into 4 groups:

Part	Processing Order and Time			
	First Machine		Second Machine	
1	M1	8	M2	10
2	M1	8	M2	8
3	M2	10	M1	11
4	M1	7	-	-
5	M2	10	-	-
6	M1	4	M2	8
7	M1	10	-	-
8	M1	4	M2	10
9	M2	9	-	-
10	M2	10	M1	8



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Type A parts: Parts 4 and 7 are to be processed on machine M1 alone.

The partial schedule  $S_a = (4, 7)$  - arbitrary order of 4 and 7.

Type B Parts: Parts 5 and 9 are to be processed on machine M2 alone.

The partial schedule  $S_b = (5, 9)$



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Type C Parts: Parts 1, 2, 6 and 8 are to be processed on machine M1 first, then on machine M2, we can set  $S_c$  using

Obtain the sorted triplets (Part Number,  $\min\{t_{i1}, t_{i2}\}$ , Corresponding Machine Number):

- (6, 4, 1)
- (8, 4, 1)
- (2, 6, 1)
- (1, 8, 1)

Part	Processing Order and Time			
	First Machine		Second Machine	
1	M1	8	M2	10
2	M1	8	M2	8
3	M2	10	M1	11
4	M1	7	-	-
5	M2	10	-	-
6	M1	4	M2	8
7	M1	10	-	-
8	M1	4	M2	10
9	M2	9	-	-
10	M2	10	M1	8

$S_c = \{6, 8, 2, 1\}$



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