`Chapter 7: Practice/review problems`

The collection of problems listed below contains questions taken from previous MA123 exams.

Max-min problems

[1]. A field has the shape of a rectangle with two semicircles attached at opposite sides. Find the radius of the semicircles if the field is to have maximum area, the perimeter of the field equals 100, and the width of the field (twice the radius of the semicircles) is at most 18. (**Caution:** Be sure your answer satisfies all conditions.) The radius equals

(a) 6 **(b)** 7 **(c)** 8 **(d)** 9 **(e)** 10

[2]. Find the area of the largest rectangle with one corner at the origin and the opposite corner in the first quadrant and on the line $y = 10 - 2x$. Assume the sides of the rectangle are parallel with the axes.

(a) 73/2 **(b)** 67/2 **(c)** 55/2 **(d)** 49/2 **(e)** 25/2

[3]. If you sell an item at price p, your revenue will equal the price p times the number sold, n. Suppose price is linearly related to the number sold by the equation

$$n = 100 - 10(p - 10)$$

How should you set the price to maximize revenue? The price should equal

(a) 10 **(b)** 15 **(c)** 20 **(d)** 25 **(e)** 30

[4]. A rectangle in the first quadrant has one corner at $(0, 0)$ and the opposite corner on the curve $y = 2 - x^2$. What is the largest possible area of this rectangle?

(a) $\frac{2}{3}\sqrt{\frac{7}{3}}$ **(b)** $\frac{8}{3}\sqrt{\frac{2}{3}}$ **(c)** $\frac{8}{3}\sqrt{\frac{4}{3}}$ **(d)** $\frac{4}{3}\sqrt{\frac{2}{3}}$ **(e)** $\frac{2}{3}\sqrt{\frac{4}{3}}$

[5]. Find the length of the shortest line segment that connects the point $(4, 0)$ in the (x, y) plane to the line $y = 2x$.

(a) $\frac{8}{5}\sqrt{5}$ **(b)** $\frac{10}{7}\sqrt{7}$ **(c)** $\frac{16}{17}\sqrt{17}$ **(d)** $\frac{12}{15}\sqrt{15}$ **(e)** $\frac{18}{19}\sqrt{19}$

[6]. Find the area of the triangle of minimum area with base equal to the unit interval $0 \le x \le 1$ on the x axis and with opposite vertex lying on the curve $y = 8x + \frac{4}{x^2}$ with $x > 0$.

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4 **(e)** 6

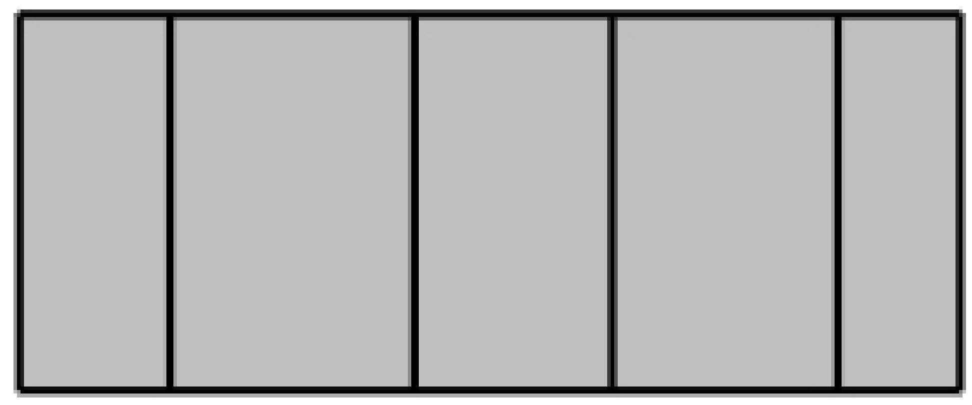
[7]. Find the area of the largest rectangle with one corner at the origin, the opposite corner in the first quadrant on the graph of the line $f(x) = 6 - 3x$, and sides parallel to the axes.

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4 **(e)** 5

[8]. What is the maximum area of the rectangle with sides parallel to the coordinate axes, one corner at the origin, and the opposite corner in the first quadrant on the ellipse given by the equation $2x^2 + y^2 = r^2$?

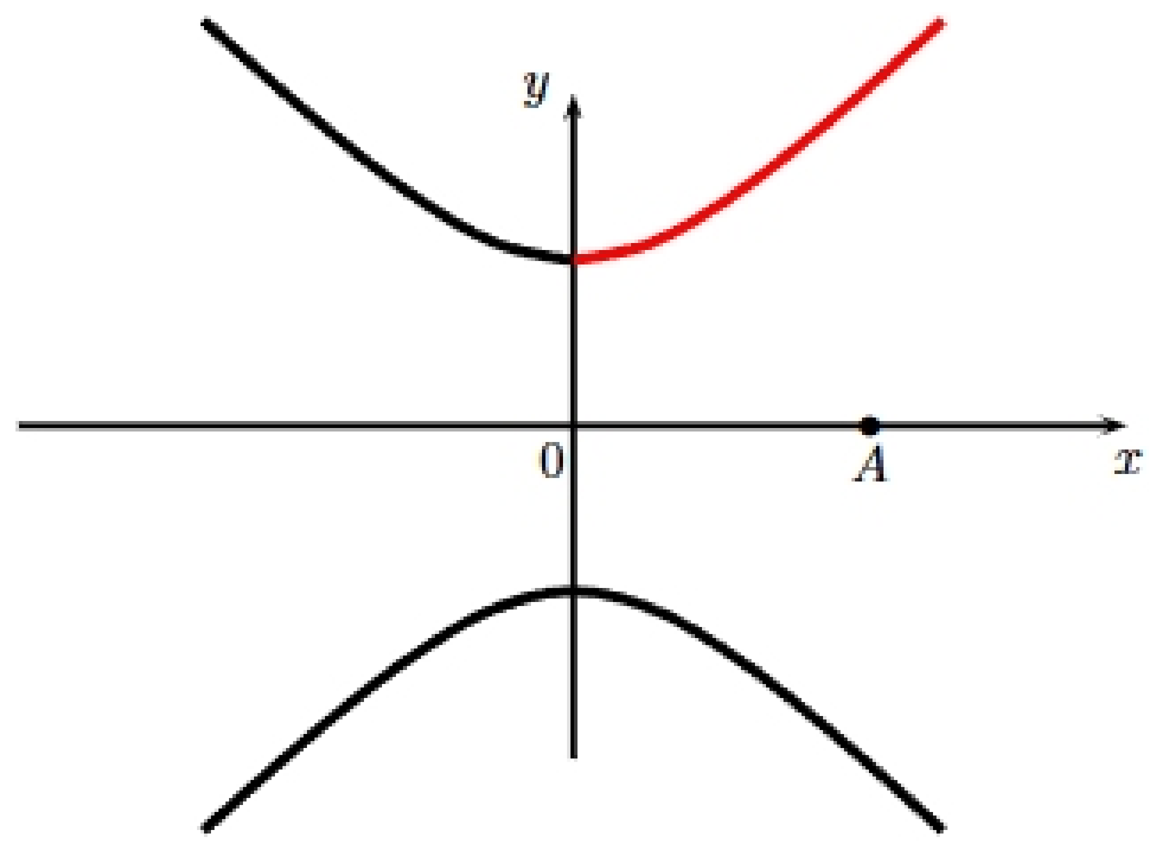
(a) r^2 **(b)** $\frac{r^2}{\sqrt{2}}$ **(c)** $\frac{r^2}{2}$ **(d)** $\frac{r^2}{2\sqrt{2}}$ **(e)** $\frac{r^2}{4}$

[9]. A rectangular field as shown below is constructed using 2400 feet of fencing. (There are six parallel fences in the vertical direction.) What is the maximum possible area in square feet of the rectangular field?



(a) 100,000 **(b)** 110,000 **(c)** 120,000 **(d)** 130,000 **(e)** None of the above

[10]. Find the point (x_0, y_0) in the first quadrant that lies on the hyperbola $y^2 - x^2 = 5$ and is closest to the point $A(4, 0)$. Then (x_0, y_0) is



(a) $(1, \sqrt{6})$

(b) $(2, 3)$

(c) $(2.5, \sqrt{11.25})$

(d) $(3, \sqrt{14})$

(e) $(4, \sqrt{21})$

[11]. Suppose you want to find the shortest distance between the point $(1, 0)$ on the x-axis and a point on the ellipse $x^2 + 4y^2 = 16$. Which problem do you need to solve?

(a) Minimize $D = \sqrt{(x-1)^2 + \left(\sqrt{\frac{16-x^2}{4}}\right)^2}$ where $-4 \leq x \leq 4$.

(b) Minimize $D = \sqrt{(x)^2 + \left(\sqrt{\frac{16-x^2}{4}} - 1\right)^2}$ where $-4 \leq x \leq 4$.

(c) Minimize $D = \sqrt{(x)^2 + \left(\sqrt{16-4x^2} - 1\right)^2}$ where $-2 \leq x \leq 2$.

(d) Minimize $D = \sqrt{(x-1)^2 + \left(\sqrt{16-4x^2}\right)^2}$ where $-2 \leq x \leq 2$.

(e) None of the above.

[12]. Suppose $y = \frac{32}{x^2}$. What is the minimum sum of x and y if x and y are both positive?

(a) 6 **(b)** 9 **(c)** 3 **(d)** 2 **(e)** 4

[13]. Suppose that the sum of x and y is 12, x and y both positive. What is the value of x that gives the largest possible value of x^2y?

(a) 6 **(b)** $\sqrt{6}$ **(c)** 8 **(d)** $\sqrt{8}$ **(e)** 4

[14]. Suppose the product of x and y is 64 and both x and y are positive. What is the minimum possible sum of x and y?

(a) 9 **(b)** 12 **(c)** 15 **(d)** 16 **(e)** 20

[15]. Find the area of the rectangle of maximum area with one vertex (corner) at $(0,0)$ and opposite corner on the ellipse $x^2 + 4y^2 = 4$.

(a) $3/4$ **(b)** $\sqrt{5}/4$ **(c)** $\sqrt{7}/4$ **(d)** 1 **(e)** $\sqrt{11}/4$

[16]. Let T be the triangle enclosed by the x-axis, the y-axis, and the line $y = 4 - 2x$. Find the area of the largest rectangle with sides parallel to the coordinate axes that can be inscribed in T.

(a) 2 square units **(b)** 8 square units **(c)** 4 square units **(d)** 6 square units **(e)** 3 square units

[17]. Let (a, b) be the point on the line $y = 4 - 2x$ that is closest to the origin $(0,0)$. What is the distance from (a, b) to $(0,0)$? (**Hint:** Draw a picture.)

(a) $2\sqrt{5}/5$ **(b)** $3\sqrt{5}/5$ **(c)** $4\sqrt{5}/5$ **(d)** $5\sqrt{5}/5$ **(e)** $6\sqrt{5}/5$

Related rate problems

[18]. At 12:00 noon a ship sailing due East at 20 miles per hour passes directly North of a lighthouse located on the coast exactly one mile South of the ship. How fast is the distance between the ship and the lighthouse increasing at 1:00 pm?

(a) $\frac{100}{\sqrt{101}}$ **(b)** $\frac{200}{\sqrt{201}}$ **(c)** $\frac{300}{\sqrt{301}}$ **(d)** $\frac{400}{\sqrt{401}}$ **(e)** $\frac{500}{\sqrt{501}}$

[19]. Water is evaporating at a rate of .5 cubic feet per day from a cylindrical tank. The circular base of the tank (parallel to the ground) has a radius of 4 feet. How fast is the depth of the water decreasing when the tank is half full (measured in feet per day)?

(a) $\frac{1}{64\pi}$ **(b)** $\frac{1}{32\pi}$ **(c)** $\frac{1}{16\pi}$ **(d)** $\frac{1}{8\pi}$ **(e)** $\frac{1}{4\pi}$