Math 217 Fall 2000 Exam 1

Notational Remark: In this exam, the symbol $\frac{\partial}{\partial x} y(x)$ means $\frac{dy}{dx}$.

1. Suppose that $y(x)$ is a solution to the differential equation $\frac{\partial}{\partial x} y(x) = F(x, y(x)), y(x_0) = y_0$. Then $y'(x_0)$ must equal:

a) y_0 b) x_0 c) $F(x_0, y(x))$ d) $F(x, y_0)$ e) $F(x, y(x))$ f) $F(x_0, y_0)$

g) $F_x(x_0, y_0)$ h) $F_y(x_0, y_0)$

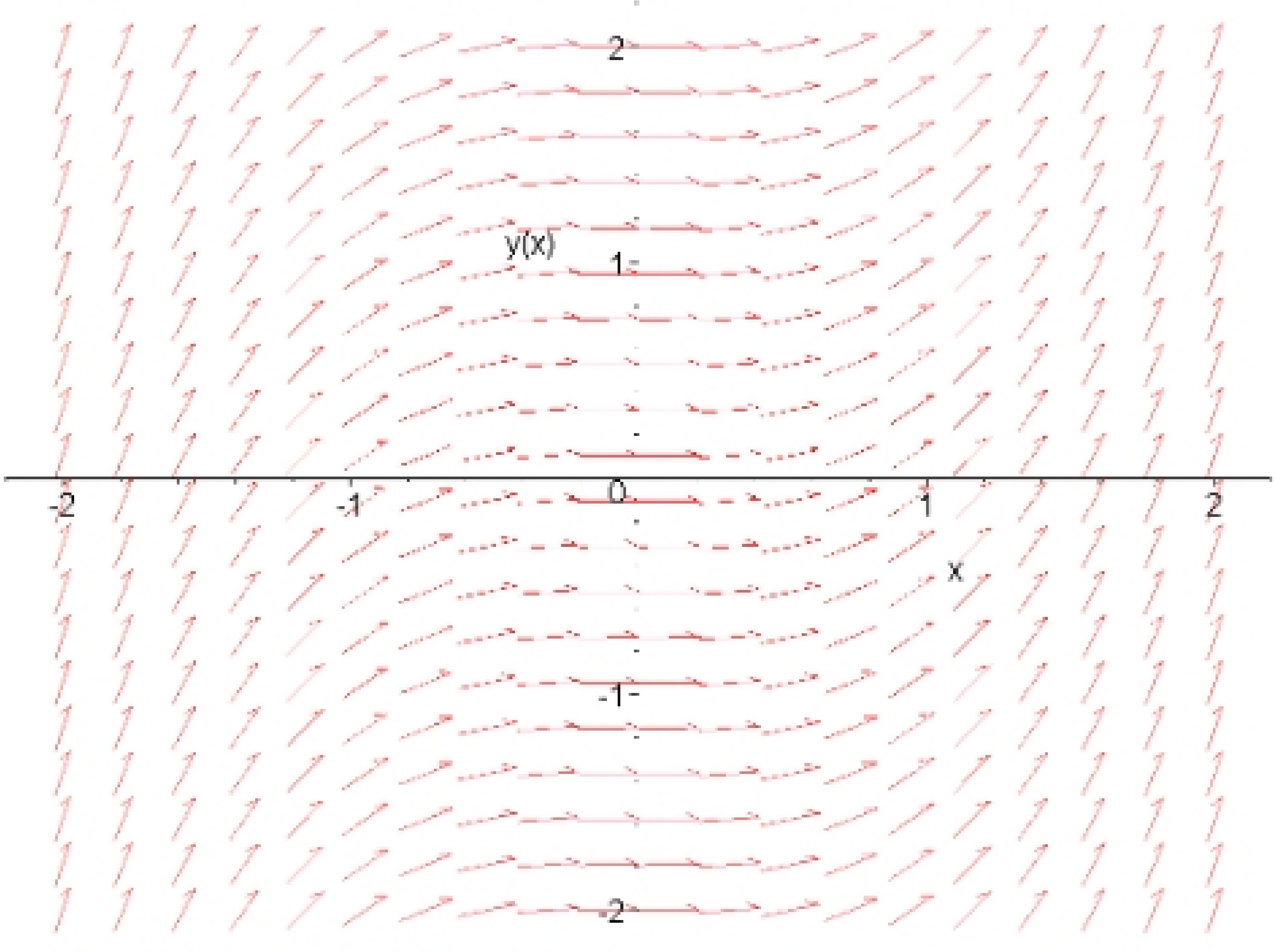
i) Can only be determined from the given information when the initial value problem is known to have a unique solution.

j) Cannot be determined from the given information.

Solution: f

$y'(x_0) = F(x_0 , y(x_0)) = F(x_0 , y_0)$

2. For which $F(x, y)$ is the direction field plot of the equation $\frac{\partial}{\partial x} y(x) = F(x, y(x))$ in the window [-2,2]x[-2,2] the following plot:



a) y^2 b) x^2 c) $x^2 - y^2$ d) $x^2 + y^2$ e) $x + y$

f) $x - y$ g) xy h) $1 - y$ i) $1 + x$ j) x^3

Solution: b

This is easiest done by elimination. The slopes evidently depend x on but not on y. That eliminates all answers but (b) and (j). All the slopes are positive. That eliminates answer (j) which is negative for negative x. Only (b) remains. With a little examination it becomes clear that the slopes in view are consistent with this answer.

3. Suppose that $y(x)$ is the solution of the initial value problem $x\left(\frac{\partial}{\partial x}y(x)\right) = 2\,y(x),\ y(1) = 3$. Then $y(2)$ is equal to:

a) 24

b) 12

c) 8

d) 6

e) 4

f) 2

g) 1

h) 0

i) -2

j) -6

<u>Solution:</u> b

The equation is separable:

$$x\left(\frac{\partial}{\partial x}y(x)\right) = 2\,y(x)$$

$$\frac{\frac{\partial}{\partial x}y(x)}{y(x)} = 2\,\frac{1}{x}$$

$$\int \frac{1}{y}\,dy = \int 2\,\frac{1}{x}\,dx + C$$

$$\ln(y) = 2\,\ln(x) + C$$

$$y(x) = e^{C}\,x^2$$

Simplify the constant and solve using $y(1) = 3$: