

Math 2270-1
Second Exam Review Information
November 2, 2005

We will use the usual problem session time and location, LCB 121, 9:40-10:30 Thursday Nov 2, for a problem session in which we will go over the practice exam.

The exam will cover section 3.4, 4.1-4.3, 5.1-5.5. In addition to being able to do the computations from these sections, you should know key definitions, the statements of the main theorems, and why they are true. The exam will be a mixture of computational and theoretical questions. As on the first exam, there will (also) be some true-false questions, see e.g. those at the ends of chapters 3, 4, 5.

One way to organize the topics is as follows:

Linear (combination) Spaces, 3.4-4.3 (also called vector spaces)

Definitions:

- Linear space
- subspace
- Linear transformation
- domain
- codomain
- kernel
- image
- rank
- nullity
- linear isomorphism
- linear combination
- span
- linear dependence, independence
- basis
- dimension
- coordinates with respect to a basis
- matrix of a linear transformation for a given basis

Theorems:

results about dimension: e.g. if $\dim(V)=n$, then more than n vectors are ?, fewer than n vectors cannot ?, n linearly independent vectors automatically ?, n spanning vectors automatically are ?

also, if a collection of vectors is dependent, it may be culled without decreasing the span; if a vector is not in the span of a collection of independent vectors, it may be added to the collection without destroying independence.

the kernel and image of linear transformations are subspaces.

rank plus nullity equals ?

A linear transformation is an isomorphism if and only if ?

Isomorphisms preserve ?

Computations:

Check if a set is a subspace

Check if a transformation is linear

Find kernel, image, rank, nullity of a linear transformation

Check if a set is a basis; check spanning and independence questions.

- Find a basis for a subspace
- Find coordinates with respect to a basis
- Find the matrix of a linear transformation, with respect to a basis
- Use the matrix of a linear transformation to understand kernel, image
- Compute how the matrix of a linear trans changes if you change bases

Orthogonality (Chapter 5)

Definitions:

- orthogonal
- magnitude
- unit vector
- orthonormal collection
- orthogonal complement to a subspace
- orthogonal projection
- angle
- correlation coefficient (not on exam, but interesting)
- orthogonal transformation, orthogonal matrix
- transpose
- least squares solutions to $Ax=b$
- inner product spaces

Theorems

- Pythagorean Theorem
- Cauchy-Schwarz Inequality
- Any basis can be replaced with an orthonormal basis (Gram Schmidt)
- Algebra of the transpose operation
- symmetric, antisymmetric
- algebra of orthogonal matrices
- Orthogonal complement of the orthogonal complement of V is V !

Computations

- coordinates when you have an orthonormal basis (in any inner product space)
- Gram-Schmidt (in any inner product space)
- $A=QR$ decomposition
- orthogonal projections (in any inner product space)
- least squares solutions
- application to best-line fit for data
- matrix for orthogonal projection
- The four fundamental subspaces of a matrix