

## Homework 22 – The Ratio and Root Tests

1) Apply the Ratio Test to determine convergence or divergence, or state that the Ratio Test is inconclusive.

a)  $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^{100}}$

b)  $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$

c)  $\sum_{n=0}^{\infty} \frac{n!}{6^n}$

d)  $\sum_{n=1}^{\infty} \frac{1}{(2n)!}$

2) Show that  $\sum_{n=1}^{\infty} n^k 3^{-n}$  converges for all exponents  $k$ .

3) Assume that  $\left[ \frac{a_{n+1}}{a_n} \right]$  converges to  $\rho = \frac{1}{3}$ . What can you say about the convergence of  $\sum_{n=1}^{\infty} n^3 a_n$ ?

4) Apply the Root Test to determine convergence or divergence, or state that the Root Test is inconclusive.

a)  $\sum_{n=1}^{\infty} \left( \frac{-3}{n} \right)^n$

b)  $\sum_{k=0}^{\infty} \left( \frac{k}{k+10} \right)^k$

c)  $\sum_{n=0}^{\infty} \left( \frac{10n}{5n+4} \right)^n$

5) Prove whether the following infinite series converge absolutely, converge conditionally, or diverge.

a)  $\sum_{n=1}^{\infty} \frac{n^3}{5^n}$

b)  $\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}}$

c)  $\sum_{n=1}^{\infty} \frac{e^n}{n!}$

d)  $\sum_{n=1}^{\infty} 4^{-2n+1}$

e)  $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n^3 - n^2}}$

f)  $\sum_{n=1}^{\infty} (-1)^n \left( \sin \frac{1}{n} \right)^n$