Two-Dimensional Rotational Dynamics

8.01
W09D2

W09D2 Reading Assignment:
MIT 8.01 Course Notes:
Chapter 17 Two Dimensional Rotational Dynamics
Sections 17.1-17.5
Chapter 18 Static Equilibrium
Sections 18.1-3

Announcements

Problem Set 7 due Week 10 Tuesday at 9 pm in box outside 26-152

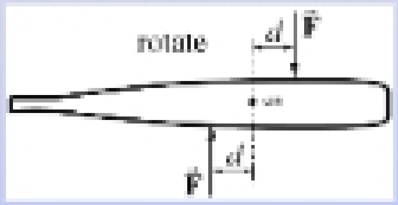
Math Review Week 10 Tuesday at 9 pm in 26-152

Exam 2 Regrade Policy

If you would like any problem regraded on your exam, please write a note on the cover sheet and submit the exam no later then the following class. Exams have been photocopied so under no circumstances should you change any of your original answers. Any altered exams will automatically be graded zero.

Rigid Bodies

- Rigid body: An extended object in which the distance between any two points in the object is constant in time.
- Effect of external forces on rigid body



Main Idea: Rotational Motion about Center of Mass

$\vec{\tau}_{cm}$ Torque about center of mass produces angular acceleration about center of mass

$$\vec{\tau}_{cm} = I_{cm}\vec{\alpha}_{cm}$$

I_{cm} is the moment of inertia about the center of mass

α_{cm} is the angular acceleration about center of mass

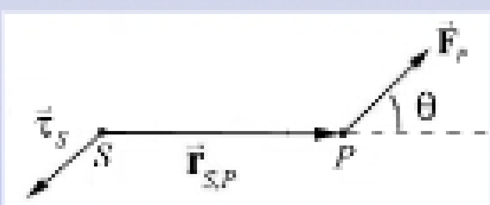
Analagous to Newton's Second Law for Linear Motion

$$\vec{\mathbf{F}}_{ext} = m\vec{\mathbf{a}}_{cm}$$

Torque as a Vector

Force $\vec{\mathbf{F}}_P$ exerted at a point P on a rigid body.

Vector $\vec{\mathbf{r}}_{S,P}$ from a point *S* to the point *P*.



Torque about point S due to the force exerted at point P:

$$\vec{\tau}_S = \vec{\mathbf{r}}_{S,P} \times \vec{\mathbf{F}}_P$$

Concept Q.: Vector (cross) Product

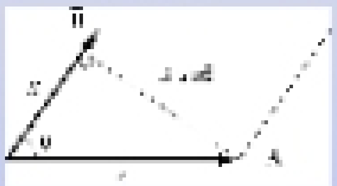
Consider a right-handed coordinate system with unit vectors $(\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}})$. The vector product $\hat{\mathbf{k}} \times \hat{\mathbf{j}}$ is equal to

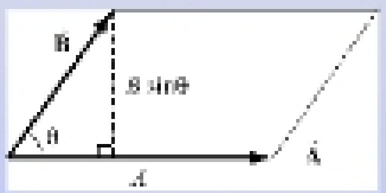
1. $\hat{\mathbf{i}}$
2. $-\hat{\mathbf{i}}$
3. $\hat{\mathbf{j}}$
4. $-\hat{\mathbf{j}}$
5. $\hat{\mathbf{k}}$
6. $-\hat{\mathbf{k}}$
7. 0
8. $\vec{\mathbf{0}}$
9. 1

Vector Cross Product

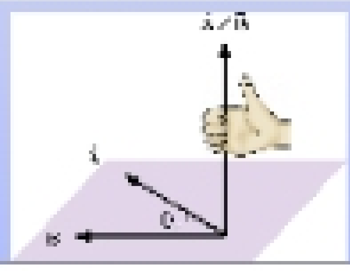
Magnitude: equal to the area of the parallelogram defined by the two vectors

$$|\vec{\mathbf{A}} \times \vec{\mathbf{B}}| = |\vec{\mathbf{A}}||\vec{\mathbf{B}}| \sin\theta = |\vec{\mathbf{A}}|(|\vec{\mathbf{B}}| \sin\theta) = (|\vec{\mathbf{A}}| \sin\theta)|\vec{\mathbf{B}}| \quad (0 \le \theta \le \pi)$$



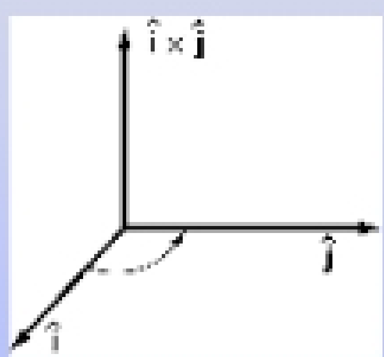


Direction: determined by the Right-Hand-Rule



Vector Product of Unit Vectors

Unit vectors in Cartesian coordinates



$$|\hat{\mathbf{i}} \times \hat{\mathbf{j}}| = |\hat{\mathbf{i}}||\hat{\mathbf{j}}| \sin(\pi/2) = 1$$

$$|\hat{\mathbf{i}} \times \hat{\mathbf{i}}| = |\hat{\mathbf{i}}||\hat{\mathbf{j}}| \sin(0) = 0$$

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \quad \hat{\mathbf{i}} \times \hat{\mathbf{i}} = \vec{\mathbf{0}}$$
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