Massachusetts Institute of Technology
Department of Mechanical Engineering

2.004 *Dynamics and Control II*
Spring Term 2008

Lecture 14[1]

Reading:

- Class Handout: *Modeling Part 1: Energy and Power Flow in Linear Systems* Sec. 3.
- Class Handout: *Modeling Part 2: Summary of One-Port Primitive Elements*

1 The Modeling of Rotational Systems.

With the the modeling framework as we defined it in Lecture 13, we have seen that in each energy domain we need to define

(a) Two power variables, an *across variable*, and a *through variable*. the product of these variables is power.

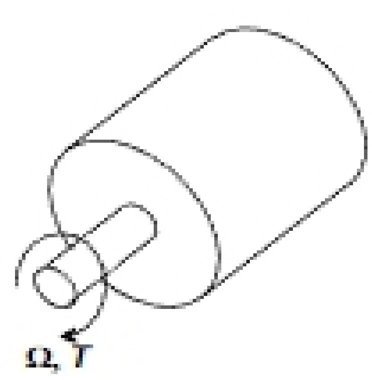
(b) Two ideal sources, and *across variable source*, and a *through variable source*.

(c) Three ideal modeling elements, two energy storage elements (a T-type element, and a A-Type element), and a dissipative (D-Type) element.)

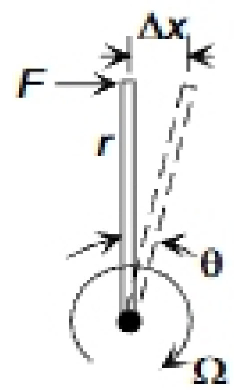
(d) A pair of interconnection laws.

We now address modeling of rotational mechanical systems.

(a) Definition of Power variables: In a rotational system we consider the motion of a system around an *axis of rotation*:



Consider the rotary motion resulting from a force F applied at a radius r from the rotational axis



The work done by the force F in moving an infinitesimal distance Δx is

$$\Delta W = F\Delta x = Fr\theta$$

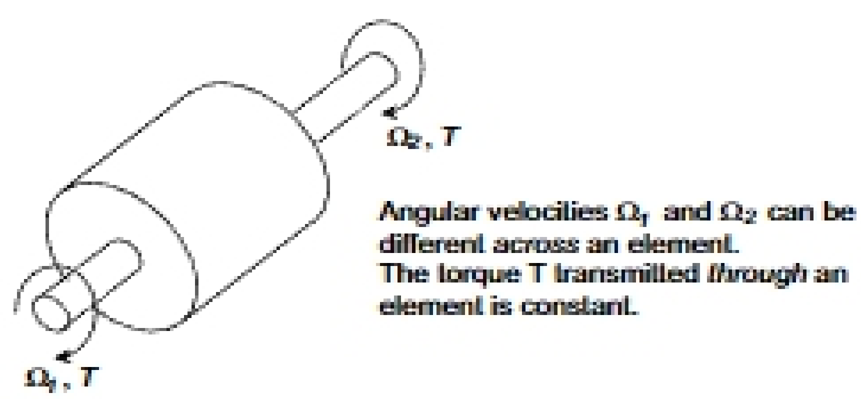
and the power P is

$$\boxed{P = \frac{\mathrm{d}\Delta W}{\mathrm{d}t} = Fr\frac{\mathrm{d}\theta}{\mathrm{d}t} = T\Omega}$$

where $T = Fr$ is the applied torque (N.m), and $\Omega = \mathrm{d}\theta/\,\mathrm{d}t$ is the angular velocity (rad/s).

We note that if T and Ω have the same sign, then $P > 0$ and power is flowing into the system or element that is being rotated. Similarly, if T and Ω have the opposite signs, then $P < 0$ and power is flowing from the system or element, in other words the system is doing work on the source.

Note that the angular velocity Ω can be different across an element, but that torque T is transmitted through an element:



> We therefore define our power variables as torque T and angular velocity Ω, where
> - T is chosen as the *through* variable
> - Ω is chosen as the *across* variable.

(b) Ideal Sources: With the choice of modeling variables we can define our pair of ideal sources

The Angular Velocity Source: $\Omega_s(t)$

By definition the angular velocity source is an *across variable source*.The ideal angular velocity source will maintain the rotational speed regardless of the torque it must generate to do so: