ECE 6130 Impedance and Admittance Matrices and S-Parameters

Text Sections: 4.2, 4.3

Describe Z and S matrices, how to compute them, and how to convert between them.
See for example Chapter 4, Problems 7,9

Impedance Matrix:

DRAW an N-port network.

Impedance matrix is used to model V and I relations for all ports.
$Z_{ij} = V_i / I_j$ with $I_k = 0$ for $k \neq j$

1) Open all other ports except j
2) Drive port j with current I_j
3) Read V_i
4) Compute Z_{ij}

$$\begin{bmatrix} V_1 \\ V_2 \\ \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & & Z_{1N} \\ Z_{21} & & & \\ & & & \\ Z_{N1} & & & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \\ I_N \end{bmatrix}$$

OR: V = Z I

Admittance Matrix:

I = Y V

$Y = Z^{-1}$ (matrices are inverses of eachother)

Reciprocal Network:

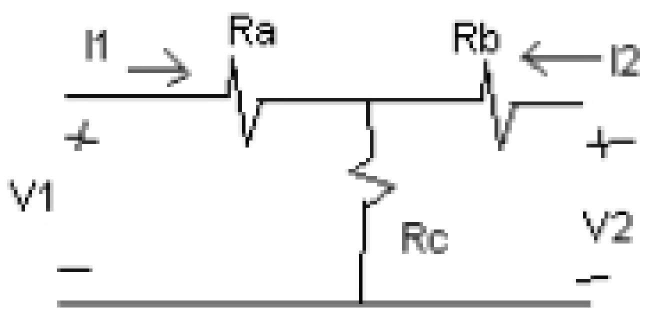
$Z_{ij} = Z_{ji}$

Examples of reciprocal networks: any R,L,C network
Examples of non-reciprocal networks: transistors, amplifiers, attenuators

Lossless Network:

Real (Z_{ij}) = 0 << Z_{ij} is strictly imaginary (change of phase, but no attentuation)

EXAMPLE: T-Network



Find Z_{11} :
I2 = 0; V1 = I1 (Ra + Rc); Z_{11} = V1 / I1 = Ra + Rc
Find Z_{12} :
I1=0; V2 = I2 (Zb + Zc); V1 = V2   Zc / (Zb+Zc); Z_{12} = V1 / I2 = Zc

Find Z_{21}:
I2 = 0; V1 = I1 (Za + Zc); V2 = V1 Zc / (Za+Zc) ; Z_{21} = V2/ I1 = Zc = Z_{12}

Find Z_{22} :
I1 =0; V2 = I2 (Zb+Zc); Z_{22} = V2 / I2 = Zb + Zc

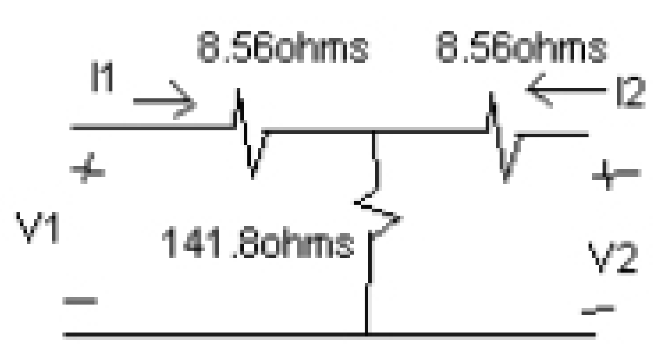
Scattering Matrix (S-parameters)

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \\ \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & & S_{1N} \\ S_{21} & & & \\ & & & \\ & & & \\ S_{N1} & & & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \\ \\ V_N^+ \end{bmatrix}$$

Where
$S_{ij} = V_i^- / V_j^+$ when $V_k^+ = 0$ for $k \neq j$

1) Terminate all ports except j with matched load.
2) Drive port j with V_j^-
3) Measure reflected voltage V_i^- on port i.

EXAMPLE: 3dB attenuator



Find S11:
Z2 = 50 ohms; Zin(port 1) = 8.56 + (141.8 || (8.56 + 50)) = 50 ohms
$V1^-$ = 0 (no reflection)
$S_{11} = V_1^- / V_1^+ = 0$

Find S22:
Circuit is symmetric.
$S_{22} = S_{11}$

Find S12: