

Probability Models

Definitions

probability model – describes all possible outcomes and says how to assign probabilities to any collection of outcomes

sample space – collection of unique outcomes of a random circumstance
– we usually denote this by S

event – a collection of outcomes

Suppose you are asked to toss a coin. What is the sample space?

You toss the coin and it lands on Tails. What is the outcome?

If you toss the coin three times and it lands THH then THH is the event.

Probability Rules

Rule 1. Any probability is a number between 0 and 1. So if we observe an event A then we know $0 \leq P(A) \leq 1$

Rule 2. All possible (disjoint or non-overlapping) outcomes together must have probability 1.

- An outcome must occur on every trial.
- The sum of the probabilities for all possible outcomes must be exactly 1.

Consider the following assignment of probabilities

Marital Status of a Random Sample of Women

Ages 25 to 29

Marital Status	Probability
Never married	0.386
Married	0.555
Widowed	0.004
Divorced	0.055

- Each of the probabilities is a number between 0 and 1.
- The probabilities total to 1.
 $0.386 + 0.555 + 0.004 + 0.055 = 1$
- *Any assignment of probabilities to all individual outcomes that satisfies Rules 1 and 2 is legitimate.*

Rule 3. The probability that an event does not occur is 1 minus the probability that the event does occur. This is known as the **complement rule**.

- Suppose that $P(A) = .70$

- Using this rule we can determine $P(\text{"not" } A)$. The event "not A" is known as the complement of A which can be written as A^c

$P(\text{"not" } A) =$

Clicker Questions about Complements and their probabilities

Suppose the probability of a horse winning a race is 0.85. What is the *complement* of the horse winning?

- A. Horse winning
- B. Horse not winning
- C. 0.85
- D. 0.15

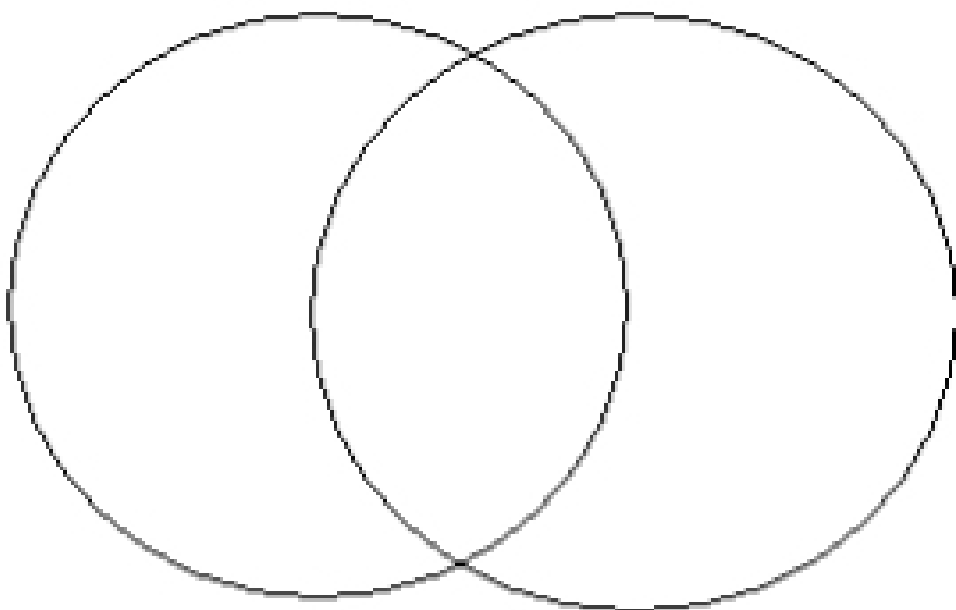
What is the *probability* of the horse not winning?

- A. Horse winning
- B. Horse not winning
- C. 0.85
- D. 0.15

Rule 4. Union Rule: If two events have no outcomes in common, the probability that one or the other occurs is the sum of their individual probabilities. If this is true then the events are said to be **disjoint**.

Suppose events A and B are disjoint and you know that $P(A) = .40$ and $P(B) = .35$. What is the $P(A \text{ or } B)$?

A Venn diagram can help us picture events and sample spaces.



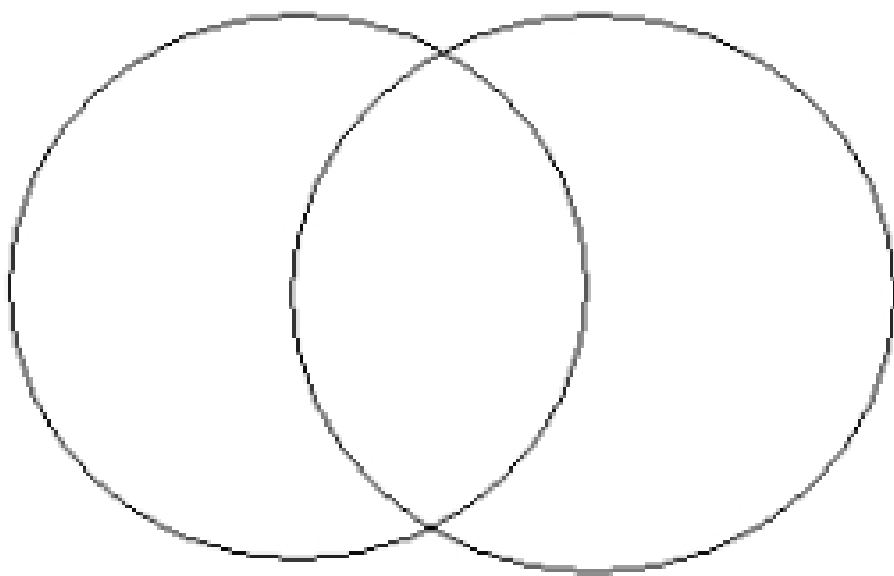
Example illustrating more general form of Rule 4

The probability a student is in honors math is 0.25, the probability a student is in honors science is 0.3, and the probability a student is in both is 0.2.

What is the probability a student is in at least one honors class?

This is called a **union** and can be pictured on a Venn diagram

Probability of honors math or science (or both) =
 $P(\text{honors math or honors science})$



The probability it will rain Wednesday AM is 30%. The probability it will rain Wednesday PM is 30%. The probability it will rain both Wednesday AM and Wednesday PM is 10%.

What is the probability it will rain on Wednesday?

- A) 20%
- B) 30%
- C) 40%
- D) 50%
- E) 60%

Rule 5. Multiplication Rule: If two events, A and B, are independent then $P(A \text{ and } B) = P(A)P(B)$

Independence means that the occurrence of event A does not affect the occurrence of event B

What is the probability that the first roll of a die is even, and the second roll is odd?

- A – 1/36
- B – 1/12
- C – 1/4
- D – 1/2