Consider the regression equation $x = \alpha + \beta y + \varepsilon$.

The least squares fit is

$$\hat{x} = a + by \text{, where, } a = \bar{x} - b\bar{y} \text{, and } b = \frac{SS_{YX}}{SS_{YY}} = \frac{\sum(y-\bar{y})(x-\bar{x})}{\sum(y-\bar{y})^2}.$$

The population mean for X can be estimated from this equation.

$$\hat{x} = \bar{x} - b\bar{y} + by$$
$$\hat{x} = \bar{x} + b(y - \bar{y})$$
$$\hat{\mu}_x = \bar{x} + b(\bar{Y} - \bar{y})$$

Therefore an estimate of the population total for X is

$$\hat{X} = N\hat{\mu}_x = N\left[\bar{x} + b(\bar{Y} - \bar{y})\right]$$
$$x''' = N\left[\bar{x} + b(\bar{Y} - \bar{y})\right]$$

<u>Regression Estimate of X</u>

$$\boxed{x''' = x' + b(Y - y')} \qquad (7.14)$$

<u>Estimated Variance of the Regression Estimate of X</u>

$$\boxed{V\hat{A}R(x''') \approx V\hat{A}R(x')(1 - \hat{\rho}_{xy}^2)} \text{, for large n} \qquad (7.17)$$

<u>Estimated Standard Error of the Regression Estimate of X</u>

From (3.9), page 63,

$$V\hat{A}R(x')(1 - \hat{\rho}_{xy}^2) = N(N-n)\frac{SS_{XX}}{n(n-1)}\left[1 - \frac{SS_{XY}^2}{SS_{XX}SS_{YY}}\right]$$

$$= \frac{N(N-n)}{n(n-1)}\left[SS_{XX}\left[\frac{SS_{XX}SS_{YY} - SS_{XY}^2}{SS_{XX}SS_{YY}}\right]\right] = \frac{N(N-n)}{n(n-1)}\left[\frac{SS_{XX}SS_{YY} - SS_{XY}^2}{SS_{YY}}\right]$$

$$= \frac{N(N-n)}{n(n-1)}\left[SS_{XX} - \left[\frac{SS_{XY}}{SS_{YY}}\right]SS_{XY}\right] = \frac{N(N-n)}{n(n-1)}\left[SS_{XX} - bSS_{XY}\right]$$

$$= \frac{n(n-2)N(N-n)}{n(n-1)}\left[\frac{SS_{XX} - bSS_{XY}}{n(n-2)}\right] = N(N-n)\frac{n-2}{n-1}\frac{SSE}{n(n-2)} = N(N-n)\frac{n-2}{n-1}\frac{MSE}{n}$$

$$\boxed{S\hat{E}(x''') = \sqrt{\frac{MSE}{n}}\sqrt{N(N-n)}\sqrt{\frac{n-2}{n-1}}} \approx \sqrt{\frac{MSE}{n}}\sqrt{N(N-n)}$$

Comparison:

If regression of x on y is a straight line through the origin, $\boxed{x = \beta y + \varepsilon}$, then $\hat{x} = by$, and $\bar{x} = b\bar{y}$. Therefore $b = \frac{\bar{x}}{\bar{y}}$.

Now, $\hat{x} = by$

$$\hat{x} = \left(\frac{\bar{x}}{\bar{y}}\right) y$$

$$\hat{\mu}_x = \left(\frac{\bar{x}}{\bar{y}}\right) \bar{Y}$$

$$\hat{X} = N\hat{\mu}_x = \left(\frac{\bar{x}}{\bar{y}}\right) Y$$

$x_{\square} = \left(\frac{\bar{x}}{\bar{y}}\right) Y = rY = x_{\square}$ $\Rightarrow$ Regression estimation = Ratio estimation when we have regression through the origin.