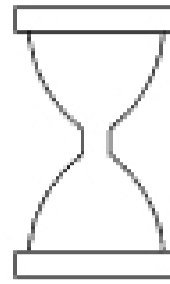


## PRODUCTION PLANNING AND SCHEDULING

### PART IV



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## MANUFACTURING SCHEDULING

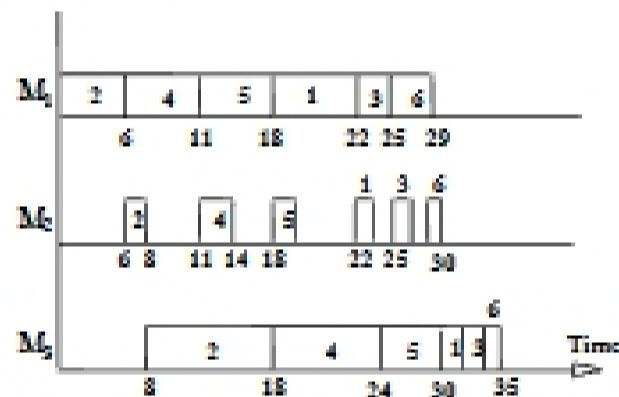
**n Jobs (Operations)**  
**m Machines**  
 + other resources



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## Example



What type of schedule is it -  
 jobshop or flowshop?



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## Scheduling Model: m Machines and n Operations

$n$  = number of parts  
 $m$  = number of machines  
 $R_i$  = set of pairs of operations  $[k, l]$  for part  $P_i$ , where operation  $k$  precedes operation  $l$ ,  $i = 1, \dots, n$   
 $Q_i$  = set of pairs of operations  $[k, l]$  for part  $P_i$ , where  $k$  and  $l$  can be performed in any order,  $i = 1, \dots, n$   
 $I_i$  = set of operations without precedence constraints,  $i = 1, \dots, n$   
 $N_p$  = set of operations to be performed on machine  $p$ ,  $p = 1, \dots, m$   
 $n_i$  = number of operations in part  $P_i$ ,  $i = 1, \dots, n$   
 $t_{il}$  = processing time of operation  $l$  of part  $P_i$ ,  $l = 1, \dots, n_i$ ,  $i = 1, \dots, n$   
 $M$  = an arbitrary large positive number



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$x_{ik}$  = completion time of operation  $k$  of part  $P_i$ ,  $k = 1, \dots, n_i$ ,  $i = 1, \dots, n$   
 $x[i]$  = completion time of the final operation of part  $P_i$ ,  
 $i = 1, \dots, n$

$y_{kl} = \begin{cases} 1 & \text{if operation } k \text{ precedes operation } l \\ 0 & \text{otherwise} \end{cases}$   
 for all  $[k, l] \in Q_i$ ,  $i = 1, \dots, n$ .

$z_{kl} = \begin{cases} 1 & \text{if operation } k \text{ precedes operation } l \\ 0, & \text{otherwise} \end{cases}$   
 for all  $[k, l] \in N_p$ ,  $p = 1, \dots, m$ .

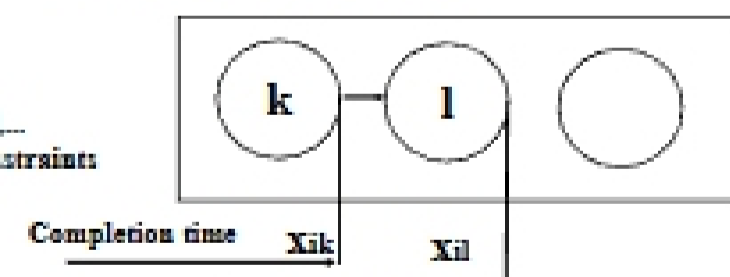


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## Basic Notation

Part  $i$ :  
 Operations  $k, l, \dots$   
 Precedence constraints:



Part  $j$



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### The Model

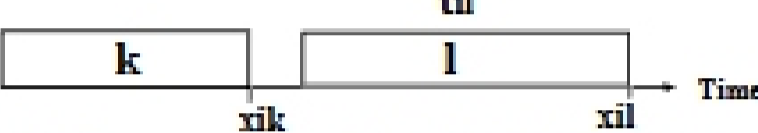
$$\text{Min } \sum_{i=1}^n x[i] \quad \text{Min the total completion time}$$



subject to:

$$xil - xik \geq til \quad \text{for all } [k, l] \in Ri, \text{ for all } i$$

Operations k and l of part i are processed according to the precedence required

Part i  $\textcircled{k} \rightarrow \textcircled{l}$

Schedule  Time

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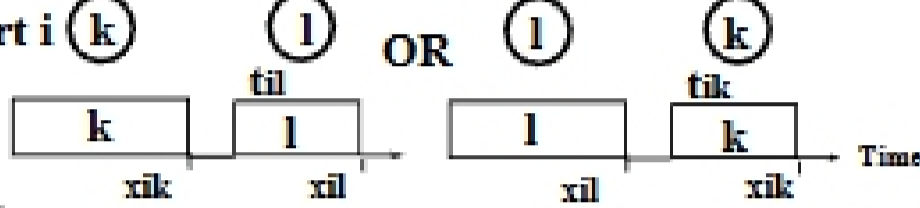
$$xil - xik + M(1 - ykl) \geq til \quad \text{for all } [k, l] \in Qi$$



$$xik - xil + Mykl \geq tik \quad \text{for all } i$$

Two operations of part i can not be processed at the same time

TWO POSSIBILITIES (Same part)

Part i  $\textcircled{k} \textcircled{l}$  OR  $\textcircled{l} \textcircled{k}$

 Time

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$$xjl - xik + M(1 - zkl) \geq tjl \quad \text{for all } [k, l] \in Np$$

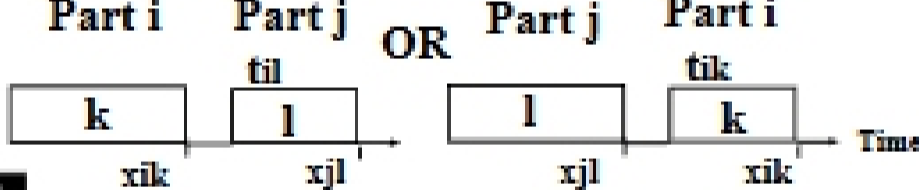
$$xik - xjl + Mzkl \geq tik \quad \text{for all } p$$



$$xik - xjl + Mzkl \geq tik \quad \text{for all } i \neq j$$

where  $Np$  = Set of operations to be performed on machine p  
A machine can not process more than one part at the same time

TWO POSSIBILITIES (Different parts)

Part i Part j OR Part j Part i

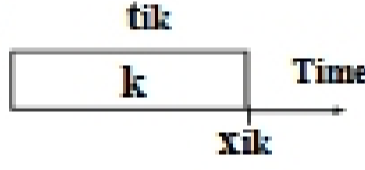
 Time

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$$xik \geq tik \quad \text{for } [i, k] \in Ii, i=1, \dots, n$$

$$0 \quad \text{for all other } i, k$$



Part i  $\textcircled{k}$

 Time

For operations without precedence constraints



$$ykl = 0, 1 \quad \text{for all } [k, l] \in Qi, \text{ for all } i$$

$$zkl = 0, 1 \quad \text{for all } [k, l] \in Np, \text{ for all } p$$

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!!!!

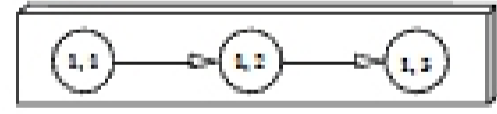
**Go over this model at least twice!**

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### Example

#### Structure of Three Parts

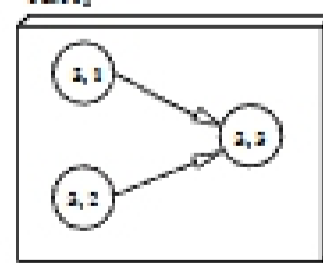
Part P<sub>1</sub>





Part P<sub>2</sub>



Part P<sub>3</sub>



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### Scheduling Data

Part number	1			2			3		
Operation number	1	2	3	1	2	3	1	2	3
Machine number	1	1	2	1	2	2	2	1	2
Processing time	3	5	6	8	4	9	2	7	

### LINDO Input File

MIN  $X_{13} + X_{21} + X_{22} + X_{33}$

SUBJECT TO

$X_{12} - X_{11} \geq 5$

$X_{13} - X_{12} \geq 6$

$X_{33} - X_{31} \geq 7$

$X_{33} - X_{32} \geq 7$

Operations of part  $i$  are processed according to precedence required

$x_{il} - x_{ik} \geq t_{il}$

Two operations of the same part  $i$  can not be processed at the same time

$x_{ik} - x_{il} + M y_{kl} \geq t_{ik}$

$x_{il} - x_{ik} + M(1 - y_{kl}) \geq t_{il}$

Derived from

$X_{22} - X_{21} + 999(1 - Y_1) \geq 4$

Part number	1			2			3		
Operation number	1	2	3	1	2	3	1	2	3
Machine number	1	1	2	1	2	2	2	1	2
Processing time	3	5	6	8	4	9	2	7	

$X_{11} - X_{21} + 999Z_1 \geq 3$

$X_{21} - X_{11} - 999Z_1 \leq -991$

$X_{11} - X_{32} + 999Z_2 \geq 3$

$X_{32} - X_{11} - 999Z_2 \leq -997$

$X_{12} - X_{21} + 999Z_3 \geq 5$

$X_{21} - X_{12} - 999Z_3 \leq -991$

$X_{12} - X_{32} + 999Z_4 \geq 5$

$X_{32} - X_{12} - 999Z_4 \leq -997$

A machine can not process more than one part at the same time

$$x_{ik} - x_{jl} + Mz_{kl} \geq t_{ik}$$

$$x_{jl} - x_{ik} + M(1 - z_{kl}) \geq t_{jl}$$

$X_{21} - X_{32} + 999Z_5 \geq 8$

$X_{32} - X_{21} - 999Z_5 \leq -997$

Part number	1			2			3		
Operation number	1	2	3	1	2	3	1	2	3
Machine number	1	1	2	1	2	2	2	1	2
Processing time	3	5	6	8	4	9	2	7	

$X_{13} - X_{32} + 999Z_6 \geq 6$

$X_{32} - X_{13} - 999Z_6 \leq -995$

$X_{13} - X_{31} + 999Z_7 \geq 6$

$X_{31} - X_{13} - 999Z_7 \leq -990$

$X_{13} - X_{33} + 999Z_8 \geq 6$

$X_{33} - X_{13} - 999Z_8 \leq -992$

$X_{22} - X_{31} + 999Z_9 \geq 4$

$X_{31} - X_{22} - 999Z_9 \leq -990$

$X_{22} - X_{33} + 999Z_{10} \geq 4$

$X_{33} - X_{22} - 999Z_{10} \leq -992$

A machine can not process more than one part at the same time

$$x_{jl} - x_{ik} + Mz_{kl} \geq t_{jl}$$

$$x_{ik} - x_{jl} + Mz_{kl} \geq t_{ik}$$

$X_{11} \geq 3$

$X_{21} \geq 8$

$X_{22} \geq 4$

$X_{31} \geq 9$

$X_{32} \geq 2$

END

Operation finish time not smaller than its processing time

INTEGER Y1

INTEGER Y2

INTEGER Z1

INTEGER Z2

INTEGER Z3

INTEGER Z4

INTEGER Z5

INTEGER Z6

INTEGER Z7

INTEGER Z8

INTEGER Z9

INTEGER Z10