

Review of Mathematical and Scientific Methods

This lab contains on-line links for review of mathematics



Objectives

<p>To review the essentials of mathematics:</p> <ul style="list-style-type: none"> • algebra • scientific notation • exponentiation (powers of 10) • logarithms • significant figures 	<p>To review the essentials of math and the scientific process as they relate to astronomy:</p> <ul style="list-style-type: none"> • small-angle formula • scaling • uncertainties in measurements • statistics • scientific method
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Materials

- 15-cm Ruler (we measure using the metric system)
- Scientific Calculator (borrow one if necessary)

Background and Theory

Mathematics is *the* language of the Universe. Imagine learning English and not being able to use any vowels:

mgn lrnng nglsh nd nt bq bl t s n vwls

or given a coded message without a key:



Similarly, astronomy cannot be taught adequately without using the proper language. Astronomy 101 covers the basic concepts in astronomy and astrophysics (really the same thing). Some concepts can be explained best and most completely by the use of a formula. You may be surprised to see how *much* information is contained in a simple algebraic equation; for example, Einstein's famous equation: $E = mc^2$. The math involved is at the level needed to be accepted into this great university. If you happen to be a '*mathophobe*' (or *arithmetically challenged*), please do not worry about the math used in this course; you will always have your fellow students and your instructor to help. You will be well prepared for any exam questions involving math.

Procedure

Each of the following steps of your review starts with a sample problem -- one that is exactly like or similar to a problem you will be seeing during this quarter. Work the problem given. If you cannot work the problem quickly, without hesitation, then take the review link offered—you will find the reviews linked through the "Math Review" link on our home page.

Students arrive at universities with a broad range of math backgrounds. It may be that you do not need much of a math review; use your talents by helping your classmates through this practice. Note, however, that the scientific methods section includes applications specific to astronomy. Every student should cover that section completely.

1. Review of Mathematics

1.1. Algebra

Solve for y : $x = 4y^2$

Solve for m (mass): $E = mc^2$

Understanding: In the equation above, E represents the energy; m , the mass; and c the speed of light -- approximately 300,000 km/sec. Why can a whole lot of energy be obtained from a tiny bit of mass?

Answer: Because the energy is the product of the mass and a very large number. The speed of light, squared, is 90,000,000,000 (km/s)²

1.2. Scientific Notation

Write the following in scientific notation: 3,042; 231.4; 0.00012; 0.0000000000667

Convert the following numbers from scientific notation:

$$4.2 \times 10^{14}$$

$$4 \times 10^{-11}$$

1.3. Exponentiation (powers of 10)

1.3.1. Work the following problems using a scientific calculator. If you do not own a calculator that uses scientific notation, then find someone who does and borrow it, or get to know them.

Multiply: 3.1×10^7 by 3×10^5

Divide: 1.496×10^{11} by 5.2×10^{-3}

Simplify: $(3 \times 10^8)^2$

Simplify: $(\sqrt{4000})^4$

1.4. Logarithms

When working with powers of 10, we will also need to reverse the process and calculate the log of a number. The log of a number tells us the equivalent number that we raise 10 to. For example: the log of 1000 is 3 because 10^3 is 1000. The log of 6000 is approximately 3.778 because $10^{3.778}$ is (about) 6000. We would have guessed that the log would be between 3 and 4 because 10^3 is 1000 and 10^4 is 10,000, and 6000 lies between those two numbers. How does one calculate the log of a number? By using a calculator. Note we will not be using the natural log, or \ln . Find the *log* button on your calculator; also, find the 10^x button to reverse the process.

Solve for M in the equation $M = m - 5 \log (d) + 5$ when:

$m =$	$d =$	Answer
2.0	10	
1.5	15	
-1.5	3	

1.5. Significant Figures

The guidelines for significant digits are:

Carry one or two non-significant digits through all calculations.

Round the final answer to the required number of significant digits.

The number of significant digits will be that of the value having the **smallest number** of significant digits.

1.5.1. How many significant digits are in the following numbers?

1.5	_____
3.5689	_____
4000	_____
3.68	_____

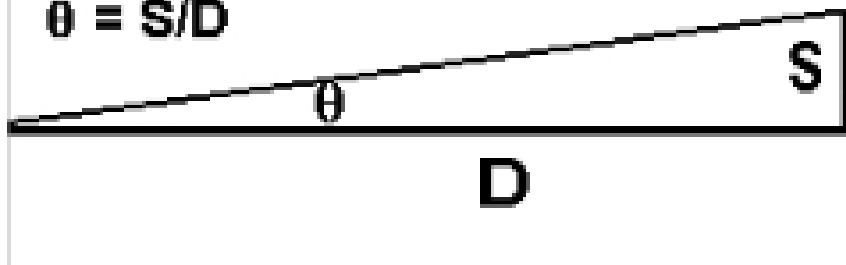
1.5.2. Multiply 1.5, 3.5689, 4000, and 3.68. Round your answer to the correct number of significant digits.

2. Astronomical and Scientific Methods

2.1. Small angle formula

For small angles:
 $\sin \theta \text{ approx.} = \tan \theta \text{ approx.} = \theta$

$$\theta = S/D$$



For small angles (much, much less than 1 degree), the sine of the angle is approximately equal to the tangent of the angle, which is approximately equal to the value of the angle itself. (Find your calculator and check this fact out. Your angle should be stated in radians.) If we know the distance to an object and can measure its angular size, we know its actual size. If we know the actual size and can measure the angular diameter, we can determine its distance.

(note: This also works if the angular size is given in arc seconds, as most measurements are in astronomy since objects such as stars and galaxies are SO far, far away. The angular size is very small!)

2.2. Practice:

Angular size in radians	Actual size in kilometers	Distance in kilometers
0.01 (diameter)	3480 (diameter)	
0.009 (diameter)		150,000,000
	6800 (diameter)	5.5×10^7