11 Low-Frequency Variability of the Sea

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11.1 Introduction

The purposeful study of the time-dependent motion of the sea having periods longer than about 1 day is comparatively recent. In the classic *Handbuch* of the early 1940s, Sverdrup, Johnson, and Fleming (1942), one searches in vain for more than the most peripheral reference to temporal changes on the large scale (one of the few examples is their figure 110 showing the California Current at two different times). Until very recently, the ocean was treated as though it had an unchanging climate with no large-scale temporal variability. The reason for this is compelling and plain: until the electronics revolution of the past 30 years, the major oceanographic observational tool was the Nansen bottle; using slow, uncomfortable ships, it took essentially 100 years to develop a picture of the gross characteristics of the mean ocean. The more recent period, 1947 (Sverdrup, 1947) through about 1970 (Stommel, 1965; Veronis, 1973b; and see chapter 5), was one of the intensive development of the theory of large-scale, steady models of the ocean circulation. The methods were initially analytic, later numerical. Most of these models were essentially low-Reynolds-number, steady, sluggish, sticky, climatic oceans. In them, the role (if any) of small-scale, time-dependent processes is simply parameterized by a positive eddy coefficient (*Austauch*) implying a down-the-mean-gradient flow of energy, momentum, heat, etc. The westward-intensification theories (Stommel, 1948; Munk, 1950) imply that any strong influence of such eddy coefficients would be confined to the western boundaries and could be ignored in the interior ocean, except possibly in the immediate vicinity of the eastward-moving free-jet "Gulf Stream" (see Morgan, 1956). The resulting models bear a remarkable resemblance to many of the gross features of the large-scale mean ocean circulation (see chapter 5).

The culmination of these analytic and numerical models of the large-scale circulation coincided with a number of developments that ultimately undermined the momentary confidence that the models represented the correct dynamics of the ocean circulation. These developments were of two kinds—instrumental and intellectual.

By 1970 instruments had been developed that made it possible to obtain *time-series* measurements in the open sea for periods far longer than a ship could possibly remain in one location. These instruments included moored current meters, drifting neutrally buoyant floats, pressure gauges, and many others (Gould, 1976; and see chapter 14). An additional "instrument" was the computer, which made it possible both to handle the large data sets generated by time-series instruments and to explore new ideas by nonanalytic means. This computer impact has been felt, of course, in most branches of science.

The intellectual developments that shifted the focus from the mean circulation to the time-dependent part were also of various kinds. The analytic models seemingly had reached a plateau at which their increasingly intricate features [e.g., essentially laminar boundary layers of higher and higher order as in Moore and Niiler (1975)] seemed untestable and intuitively implausible outside the laboratory. Physical oceanography is also to some extent a mirror of meteorology; by 1970 most oceanographers were at least vaguely familiar with the picture of the atmosphere that had emerged over the previous decades. In that fluid system, the view of the role of eddies had shifted from a passive means of dissipating the mean flows (through purely down-gradient fluxes of momentum, energy, etc.) to a much more interesting and subtle dynamic linkage in which the mean flows (the climate) were in at least some parts of the system *driven* by the eddy fluxes (Jeffreys, 1926; Starr, 1968; Lorenz, 1967). Because many of the meteorological results would apply to any turbulent fluid, there was reason to believe that the ocean could also exhibit such intimate dynamic linkages. But we should note that even now much work is still directed at studying the mean circulation by essentially classical (though improved) means, as if the variability were not dynamically important (e.g., Schott and Stommel, 1978; Wunsch, 1978a; Reid, 1978). The extent to which such pictures of the mean circulation of the large-scale tracers will survive complete understanding of variability dynamics is not now clear.

In this chapter we shall review what is known about the variability of the ocean. The expression "low-frequency variability," which is part of the title of this chapter, is a vague one used in a variety of ways by oceanographers, and encompassing a wide range of things. Here we mean by it anything with a time scale longer than a day out to the age of the earth, although we cannot really study by instrumental means phenomena with time scales longer than about 100 years. In spatial scale, it means phenomena ranging from some tens of kilometers to the largest possible global ocean oscillations. We shall, in common with recent practice, also refer to the "eddy" field in the ocean. This word is often prefixed by "mesoscale" and is used loosely to denote the subclass of variability encompassing motions occurring on scale of hundreds of kilometers with time scales of months and longer. It is a convenient shorthand and is meant to imply neither any particular dynamics nor only flows with closed streamlines. (The equivalent Soviet term is "synoptic scale".)

There is little doubt that oceanographers were quite aware, from the very beginning, of time variability in the ocean. Maury (1855, p. 358) remarked that in drawing his charts he had disregarded "numerous eddies and local currents which are found at sea." He also notes in particular (p. 188) the highly variable equatorial currents of the Pacific Ocean. Even earlier, Rennel (1832) had quoted another observer (C. Blagden), as referring to North Atlantic currents as "casual" (Swallow, 1976).

Most of the astute observers who worked at sea since Maury were very conscious of the difficulties of drawing conclusions about the mean circulation in the presence of a highly time-dependent field. Figure 11.1, taken from Helland-Hansen and Nansen (1909), clearly depicts what one suspects to be a time-dependent eddy field. Sverdrup et al. (1942) make the statement that determining the mean is difficult in the presence of the time variations, and that the closer the station pairs are together, the greater is the requirement of simultaneity in hydrographic measurements. This is, of course, a statement about the frequency-wavenumber character of the baroclinic variability.

It is possible to give many instances of references to ocean variability and eddies throughout the history of observational oceanography. But it is also fair to say

Figure 11.1 Chart of Norwegian Sea surface currents as constructed by Helland-Hansen and Nansen (1909); reproduced by Sverdrup et al. (1942). One presumes the small-scale currents are in fact time-dependent features.

that little attention was paid to the phenomenon per se; it was a nuisance—a noise-contaminating determination of the time mean flow. There are some major exceptions, including Pillsbury's (1891) heroic efforts in the Florida Current, the 400-page work by Helland-Hansen and Nansen (1920), and somewhat later, Iselin's (1940a) attempts at monitoring the western North Atlantic. The question of the physical significance of a weak mean flow in the presence of strong variability has rarely been addressed even now.

11.1.1 Early Theory

The first theoretical attempts to study the purely time-dependent oceanic motions at low frequency seem to be outgrowths of the papers by Rossby and collaborators (1939) and by Haurwitz (1940a). These two studies, while directed primarily at the atmosphere, nonetheless addressed themselves to the large-scale time-dependent wave motions of a rotating hydrostatic fluid—a characterization applying equally well to the ocean. The Rossby paper in particular introduced the *β-plane* approximation. These early efforts, and the large number that followed, examined the wave motions known much earlier. Indeed Laplace (1775) had discussed motions that we now would call Rossby or planetary waves (or in Hough's terminology, "tidal motions of the second class"). The study of these motions has a long and distinguished history (e.g., Darwin, 1886; Rayleigh, 1903; Poincaré, 1910), culminating in Hough's (1897, 1898) remarkable study of the solutions of the Laplace tidal equations on a sphere. [Lamb (1932), in his chapter on tides gives a good summary of this work. He also thoroughly discusses (§206 and §212) what we call "topographic Rossby waves" in which topographic gradients play a role analogous to the variation of the Coriolis parameter with latitude on a sphere.]

But it was Rossby's β-plane that demonstrated the physics in the simplest form and permitted an escape from the geometrical complexities of spherical coordinates. Arons and Stommel (1956), Veronis and Stommel (1956), Rattray (1964), Rattray and Charnell (1966), and others made explicit attempts to understand the possible role of Rossby waves in the ocean. Longuet-Higgins in a series of papers (1964, 1965) justified the β-plane approximation and carried out a modern exhaustive search of the solutions on a sphere for a complete range of parameters far beyond what Hough could do in his time (Longuet-Higgins, 1968a, Longuet-Higgins and Pond, 1970). Most of this work was done in the absence of any direct observational base in the ocean. (For further discussion of these waves, see chapters 10 and 18).

Observations, which will be discussed at length below, suggest that linear wave models are inadequate to describe much of the actual time-dependent motion in the ocean. Nonetheless, as in the atmosphere (Holton, 1975), many of the features of the observations are qualitatively similar to those deducible from the linear theories. That is, the physics is modified by the nonlinearity, but many of the linear features persist into the nonlinear range. The precise extent to which this is true is a function of the periods and spatial scales of the motions and is not really understood. As a generalization, it may be safe to assert that the largest oceanic scales of fluctuation are most likely to be dominated by linear dynamics. A linear description becomes increasingly doubtful for smaller scales, and barotropic motions ought to be more nearly linear than baroclinic ones (Rhines, 1979).

The postulate of a time-dependent field in the interior ocean immediately calls into question (Stommel, 1965, p. 221) one of the fundamental deductions of the steady-ocean models—that a Sverdrup balance applies in the interior ocean.

Consider, for example, Stommel's (1948) model of a homogeneous flat-bottom ocean on a β-plane. Let ψ_0 be the time-mean transport streamfunction and let ψ_1 be the time-dependent part. Then the time average vorticity balance may be written

$$J(\nabla^2\psi_0, \psi_0) + \langle J(\nabla^2\psi_1, \psi_1)\rangle + \beta\frac{\partial\psi_0}{\partial x} + R\,\nabla^2\psi_0 = -\hat{\mathbf{k}}\cdot\nabla\times\tau, \qquad (11.1)$$

where the bracket denotes a temporal average, $\hat{\mathbf{k}}\cdot\nabla\times\tau$ is the vertical component of the mean wind-stress curl, J denotes the Jacobian operator, and R is the coefficient of bottom friction. Let us assume that the mean field varies over scales of 10^4 km, and that the time-dependent eddy field varies over 10^2 km. Let both the mean flows and the time-dependent part have magnitude $10\ \mathrm{cm\,s^{-1}}$. Scaling, we obtain roughly, in nondimensional form,

$$10^{-4}\,\nabla^2\psi_0 + \frac{\partial\psi_0}{\partial x} + 10^{-3}J(\nabla^2\psi_0, \psi_0) + 10^{1}\langle J(\nabla^2\psi_1, \psi_1)\rangle = -\hat{\mathbf{k}}\cdot\nabla\times\tau. \qquad (11.2)$$

Away from the western wall, the first term is negligible; hence if we ignore both nonlinear terms, an interior balance is

$$\frac{\partial\psi_0}{\partial x} = -\hat{\mathbf{k}}\cdot\nabla\times\tau, \qquad (11.3)$$

which is the conventional Sverdrup balance. But the nonlinear term

$$10^{1}\langle J(\nabla^2\psi_1, \psi_1)\rangle \qquad (11.4)$$

will be of the same order as the Sverdrup terms if the