

(Notes from Elizabeth Weaver.)

### 12.5.1 The Method of Sealed Bids

- The method of sealed bids can be used in fair division problems with any number of players where the set  $S$  is discrete.
- **The Method of Sealed Bids:**
  1. **Bidding:** Each player makes a bid (in dollars) for each of the items in the set  $S$  and places it in a sealed envelope to maintain the privacy condition.
  2. **Allocation:** Each item will go to the highest bidder for that item. (Ties can be broken with a coin flip.)
  3. **First Settlement:** First calculate each player's fair share of the estate. If the items a player received in step 2 total more than his fair share, he must pay the difference in cash. If the items a player received in step 2 total less than his fair share, he receives the difference in cash.
  4. **Division of the Surplus:** Calculate the difference between the amount the players paid and the amount the players received in step 3. This amount is the surplus, and it is to be divided among all of the players equally.
  5. **Final Settlement:** This is obtained by adding each player's share of the surplus money to the first settlement obtained in step 3.
- **Example:** In her last will and testament, Granny leaves her estate to Jed, Elly May, Jethro and Mr. Drysdale with the stipulation that it must be divided equally and remain with them (not be sold to outsiders). Below is a listing of each player's bids on the items.

	Jed	Elly May	Jethro	Mr. Drysdale
Cabin	\$220,000	\$250,000	\$211,000	\$198,000
Rolls Royce	\$40,000	\$30,000	\$47,000	\$52,000
Picasso	\$280,000	\$240,000	\$234,000	\$190,000
Total				
Fair Share				

- (a) Who won which items?
  - (b) How much should each person receive from (or pay to) the estate?
  - (c) How much surplus is there? How much does each person receive of the surplus?
  - (d) Describe which players get what items (and how much money they pay or receive) in the final settlement.
- What are the disadvantages of the method of sealed bids?

- **Example:** Chuck and Sarah are getting a divorce. The only joint property (of value) they own is their house. They decide to use the method of sealed bids. Chuck bids \$140,000 on the house, Sarah bids \$164,000. Sarah bid higher and therefore wins the house. She pays her fair-share into the estate, while Chuck receives his in cash.

(a) How much is each person's fair-share?

(b) How much is the surplus?

(c) Describe the final settlement.

- **Example:** Elmo, Grover, and Bert are dividing four pieces of furniture using the method of sealed bids. Their bids on each of the items are given in the following table.

	Elmo	Grover	Bert
Dresser	\$150	\$300	\$275
Desk	\$180	\$150	\$165
Vanity	\$170	\$200	\$260
Table	\$400	\$250	\$500

(a) What is the value of each person's fair share?

(b) How much should each person receive from (or pay to) the estate?

(c) How much surplus is there? How much does each person receive of the surplus?

(d) Describe which players get what items (and how much money they pay or receive) in the final settlement.

- **Example:** Jack, Janet, and Chrissy jointly own a house. They no longer want to be roommates and decide to use the method of sealed bids, with the understanding that one of them will get the house and the other two will get cash. Jack bids \$180,000, Janet bids \$120,000, and Chrissy bids \$150,000. After the final settlement, how much money do Janet and Chrissy get from Jack for their one-third share of the house?

### 12.5.2 After this lecture, you should be able to...

- carry out the method of sealed bids.