Section 2.3
Solving Systems of Linear Equations II

In the previous section we studied systems with unique solutions. In this section we will study systems of linear equations that have infinitely many solutions and those that have no solution. We also will study systems in which the number of variables is not equal to the number of equations in the system.

A System of Equations with an Infinite Number of Solutions

Example 1: The following augmented matrix in row-reduced form is equivalent to the augmented matrix of a certain system of linear equations. Use this result to solve the system of equations.

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

A System of Equations That Has No Solution

Example: Given the following system

$$\begin{aligned} x + y + z &= 1 \\ 3x - y - z &= 4 \\ x + 5y + 5z &= -1 \end{aligned}$$

In using the Gauss-Jordan elimination method the following equivalent matrix was obtained (note this matrix is not in row-reduced form, let's see why):

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -4 & -4 & 1 \\ 0 & 0 & 0 & -1 \end{array}\right)$$

Look at the last row. It reads: $0x + 0y + 0z = -1$, in other words, $0 = -1$!!! This is never true. So the system is inconsistent and has no solution.

Systems with No Solution

If there is a row in the augmented matrix containing all zeros to the left of the vertical line and a nonzero entry to the right of the line, then the system of equations has no solution.

Theorem

I. If the number of equations is greater than or equal to the number of variables in a linear system, then one of the following is true:

a. The system has no solution.
b. The system has exactly one solution.
c. The system has infinitely many solutions.

II. If there are fewer equations than variables in a linear system, then the system either has no solution or it has infinitely many solutions.

Example 2: Solve the system of linear equations using the Gauss-Jordan elimination method.

$$x + 2y - 3z = -2$$
$$3x - y - 2z = 1$$
$$2x + 3y - 5z = -3$$

Example 3: Solve the system of linear equations using the Gauss-Jordan elimination method.

$$2x + 3y = 2$$
$$x + 3y = -2$$
$$x - y = 3$$

Example 4: Solve the system of linear equations using the Gauss-Jordan elimination method.

$$4x + y - z = 4$$
$$8x + 2y - 2z = 8$$

Example 5: Given that the augmented matrix in row reduced form. First we need to determine if this system has a solution then if it has a solution is to find that solution or solution(s).

a. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right]$

b. $\left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 0 & 2 \end{array}\right]$

c. $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$