EE 428 **EXAM II** **14 November 2005**

Last Name (Print): Solutions

First Name (Print): ____________

ID number (Last 4 digits): ____________

Section: ____

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

Problem	Weight	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

INSTRUCTIONS

1. You have 2 hours to complete this exam.
2. This is a closed book exam. You may use one 8.5" × 11" note sheet.
3. Calculators, protractors, and rulers are allowed.
4. Solve each part of the problem in the space following the question. If you need more space, continue your solution on the reverse side labeling the page with the question number; for example, **Problem 1.2 Continued. NO** credit will be given to solutions that do not meet this requirement.
5. **DO NOT REMOVE ANY PAGES FROM THIS EXAM.** Loose papers will not be accepted and a grade of **ZERO** will be assigned.
6. The quality of your analysis and evaluation is as important as your answers. Your reasoning must be precise and clear; your complete English sentences should convey what you are doing. **To receive credit, you must show your work.**

Problem 1: (25 Points)

1. (7 points) The feedback control system in Figure 1 has a single adjustable parameter K. Specify the transfer function $G(s)$ in the characteristic equation

$$1 + KG(s) = 0$$

of the closed-loop system.

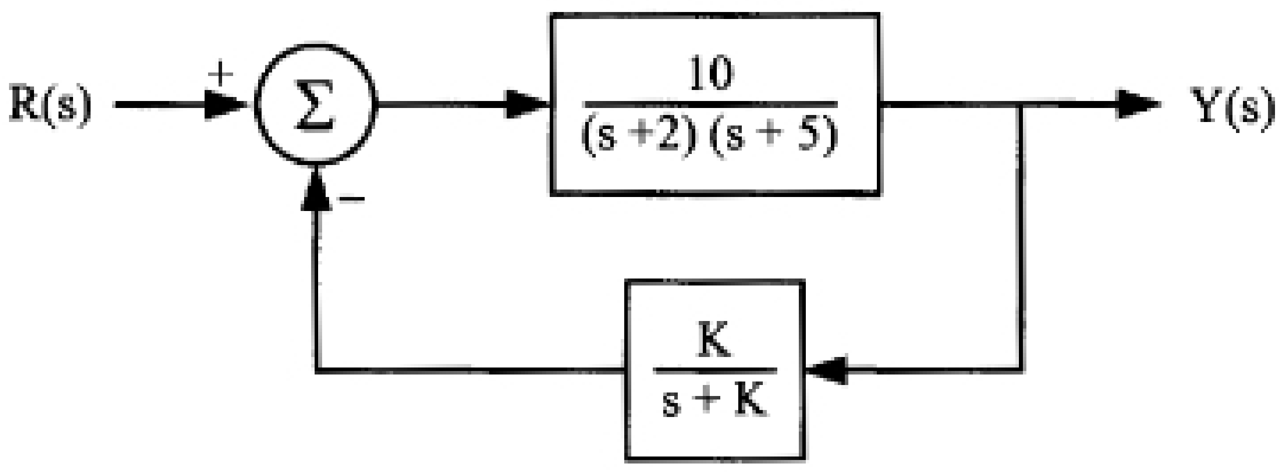

Figure 1: Closed-loop system with adjustable controller gain K.

$$\frac{Y(s)}{R(s)} = \frac{G}{1 + GH} = \frac{10(s+K)}{(s+2)(s+5)(s+K) + 10K}$$

Characteristic equation:

$$(s+2)(s+5)(s+K) + 10K = 0$$

$$s^3 + 7s^2 + 10s + K(s^2 + 7s + 20) = 0$$

$$1 + K\frac{s^2 + 7s + 20}{s(s^2 + 7s + 10)} = 0$$

$$\boxed{G(s) = \frac{s^2 + 7s + 20}{s(s^2 + 7s + 10)}}$$

2. (18 points) Consider the closed-loop system in Figure 2 with reference input $R(s)$, controlled output $Y(s)$, plant transfer function

$$G_p(s) = \frac{(s+1)^2}{s^3}$$

and proportional control

$$G_c(s) = K.$$

Sketch the root locus of the closed-loop system in Figure 3 as the controller gain varies from zero towards infinity (an extra copy is provided in Figure 4). When applicable, specify break away and break in points, asymptotes and their real axis intercept, arrival and departure angles, and imaginary-axis crossings. Use arrows to indicate the direction of travel along the loci as K approaches infinity.

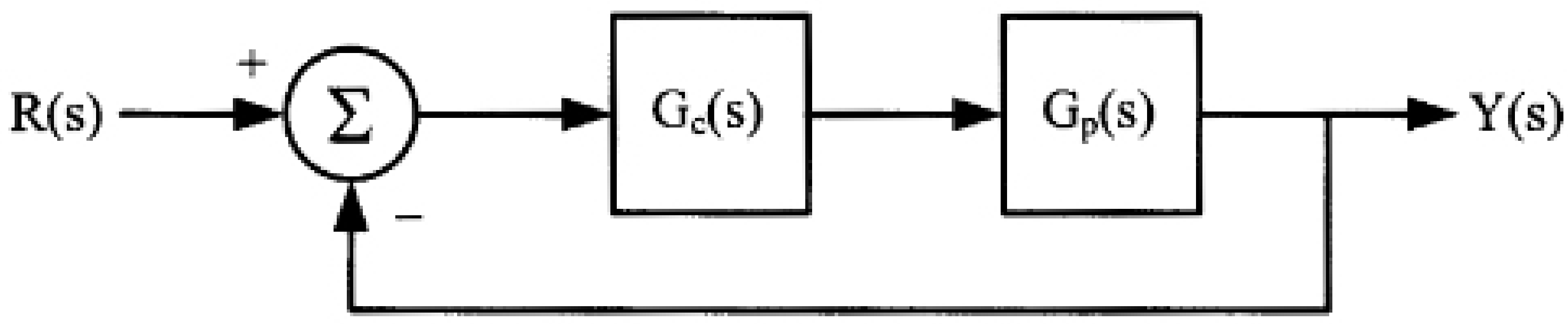

Figure 2: Closed-loop system with cascade compensation.

- On the real axis, the root locus lies between the three poles at $s=0$ and two zeros at $s=-1$, as well as to the left of the two zeros.

- For two poles to reach the two zeros at $s=-1$, there must be break in and break away points:

$$-\frac{d}{ds}\frac{1}{G(s)} = -\frac{2(s+1)s^3 - 3s^2(s+1)^2}{(s+1)^4} = 0$$

$$s^2\{2s^2+2s-3s^2-6s-3\} = s^2(s^2+4s+3) = s^2(s+1)(s+3) = 0.$$

The break away point must be at $s=0$, while the break in point $s=-3$, must lie to the left of the zeros at $s=-1$.

- Angle departure: $2\angle\underbrace{(s_t-z)}_{0^\circ} - 3\angle\underbrace{(s_t-p)}_{\phi_d} = 180+360\ell$; with s_t located near $s=0$. And so $\phi_d = \pm 60°$.

- $j\omega$ axis crossings:

$$1 + KG(s) = 0 \Rightarrow s^3 + K(s+1)^2 = s^3 + Ks^2 + 2Ks + K = 0$$

s^3	1	$2K$
s^2	K	K
s^1	$2K-1$	0
s^0	K	

set $K=\frac{1}{2}$ to obtain row of zeros. The auxiliary eqn,

$$s^2+1=0,$$

reveals that the $j\omega$-axis crossing occurs at $\pm j$.