

## Review for Exam II

You should know how to apply all of the following problem solving strategies:

1. Guess and check
2. Tables
3. Lists
4. Patterns
5. Algebra
6. Work backwards
7. Eliminate possibilities
8. Pigeon hole principle

You should know all of the following definitions

- Polya's principle
- Inductive reasoning
- Deductive reasoning
- Rule of direct reasoning
- Rule of indirect reasoning

You should know all of the following number theoretic definitions

- Divisibility
- Factor
- Multiple
- Greatest common divisor
- Least common multiple

You should know the divisibility tests for 2,3,4,5,7,8,10,11, and 13. You should be able to argue that if  $a$  and  $b$  are divisible by  $c$  then  $a + b$ ,  $a - b$ , and  $a \cdot b$  are all divisible by  $c$ .

Example Problems:

1. Polya's Principles
  - (a) State Polya's four principles.
  - (b) With one or two sentences each, explain these four principles.
2. Solve the following problem using the four Polya principles. You should indicate where and how you are applying each step of the principles. Write up your answer as if you were presenting it to a classroom. That is, your write up should be brief (one or two sentences per step) and well organized.

**Problem.** Find the  $n^{\text{th}}$  term of the sequence: 3, 5, 7, 9, 11, 13, ...

3. Solve the following problem using the four Polya principles. You should indicate where and how you are applying each step of the principles. Write up your answer as if you were presenting it to a classroom. That is, your write up should be brief (one or two sentences per step) and well organized.

**Problem.** It takes Tom 4 hours to clean the garage and it takes Maria 3 hours. If they work together, how long will it take?

4. Solve the following problem using the four Polya principles. You should indicate where and how you are applying each step of the principles. Write up your answer as if you were presenting it to a classroom. That is, your write up should be brief (one or two sentences per step) and well organized.

**Problem.** What is the sum of all the numbers in the 20th row of pascals triangle? How many numbers are in this row? Solve the problem without writing down this row.

5. (a) State the pigeonhole principle.  
(b) If there are 367 people in the education department, explain why at least two of them share a birthday.  
(c) If there are 250,000 people in Lexington and any person can have at most 200,000 hairs on their head, explain why you can conclude that a least two people will have the same number of hairs on their head.
6. Mathematical Reasoning
- (a) Define inductive and deductive reasoning. Compare and contrast these two ideas.  
(b) We solved many types of problems using the pattern recognition strategy. What type of mathematical reasoning do you use in this strategy? Explain.  
(c) Which type of reasoning does a detective typically use? Explain.  
(d) Suppose that you know  $a \rightarrow b$ . If  $b$  is false what can you conclude and why?  
(e) Suppose that  $a \rightarrow b$  is true and that  $b$  is true. Is  $a$  true? Explain and give an example.
7. If  $a$  and  $b$  are whole numbers both divisible by  $c$  prove that  $a + b$  is also divisible by  $c$ . (Hint: First, write down what it means for  $a$  and  $b$  to be divisible by  $c$ )
8. Show that the product of an even and an odd number must be an even number. (Hint: Use the useful representations of even and odd numbers.)
9. Is the number 36, 335, 936, 637 divisible by 7, 13, or 11? Argue with the divisibility tests and show all of your work.
10. (a) Give the definition for the greatest common divisor of the numbers  $a$  and  $b$ .  
(b) Give the definition for the least common multiple of the numbers  $a$  and  $c$ .  
(c) Find the greatest common divisor of the numbers 1092 and 525.  
(d) Find the least common multiple of the numbers 1092 and 525.
11. Suppose that  $n = a \cdot b$  and that the number  $c$  divides  $n$ . Is it always true that  $c$  divides  $a$  or  $b$ ? Explain your answer.
12. Consider the number  $abc, abc$  where the letters  $a, b, c$  represent digits. Argue that this number is divisible by 7, 11, and 13.
13. Mark the following as true or false. Explain each answer with one sentence, giving examples where appropriate.
- (a) Every whole number can be written as a product of prime powers.  
(b) An even number times an even number is always odd.  
(c) If the statement "if  $p$  then  $q$ " is a true conditional statement and " $q$ " is true then we may conclude that " $p$ " is true.  
(d) If the statement "if  $p$  then  $q$ " is a true conditional statement and " $p$ " is true then we may conclude that " $q$ " is true.  
(e) One way to find the least common multiple of  $a$  and  $b$  is to just multiply  $a$  and  $b$  together.