Exercises #2
due: Tuesday, January 29

Reading Assignment:

- For additional information on this week's material, please see
  Cooper, *A Matlab Companion ...*, Chapters 2.2, pp 22–24
  Thornton and Rex *Modern Physics*, Section 12.6
  Giordano *Computational Physics* Chapter 1 and Appendix A1.1, A1.2
- To prepare for next week, please read
  Giordano *Computational Physics* Chapter 3.1 pp 42–48
  in your textbook for Elementary Classical Physics:
  Giancoli, Chapter 14, Oscillations
  Fishbane, Chapter 13, Simple Harmonic Motion
  Reese, Chapter 7, Hooke's Force Law and Simple Harmonic Oscillation
  Please focus on simple harmonic motion (mass and spring, the simple pendulum) and damped oscillations.

1. Nuclear Decay (single ODE with initial value):

   (a) Open the files *ode1nuc.m*, *f1nuc.m*, and *exactnuc.m* in the editor and familiarize yourself with their contents. Copy the file ode1nuc.m into your word document and indicate in your document (by changing the type face to boldface, for example)

      i. where the initial value of y is set
      ii. where the step size is determined
      iii. where the function *exactnuc.m* is called
      iv. where the function *f1nuc.m* is called
      v. where the errors of the numerical methods are calculated

   (b) Run the program and look at the graphs. Comment on the accuracy of the two methods.

   (c) In class we discussed the effect of step-size on the precision of the method. How can you change the step-size in this program? Change the parameters so that you have a step size that is smaller by a factor of five and run the program again. What happens to the relative errors?

2. Bacterial growth (single ODE with initial value): see Giordano, Problem 6, p. 14
   Population growth problems often give rise to rate equations that are first order. For example, the equation

$$\frac{dN}{dt} = aN - bN^2, \tag{1}$$

where a and b are positive constants, might describe how the number of individuals in a population, N, varies with time. Here the first term corresponds to the birth of new members while the second term corresponds to deaths. The death term is proportional to N^2 to allow for the fact that food will become harder to find when the population becomes large.

(a) On paper:
Calculate the exact solution to equation (1)

(b) Change the mfiles from the first problem in such a way that the Runge-Kutta method is used to solve the differential equation (1) for an initial population $N(t = 0) = N_0$.

(c) From your intuition and from the exact solution, what behavior do you expect for $N(t)$ for the case $b = 0$ (remember $a > 0$). Run the program for this case, compare your numerical results with the exact solution. Does $N(t)$ behave the way you expected? If not, why not?

(d) Modify your program to solve the differential equation (1) with $a = 10$ and $b = 3$ for an initial population $N(t = 0) = N_0$. Give an intuitive explanation for your results and compare with the exact solution.

3. On paper:
Damped harmonic motion
In general, the differential equation for a damped harmonic oscillator is given by

$$\frac{d^2y}{dt^2} + 2\gamma\frac{dy}{dt} + \Omega^2 y = 0 \tag{2}$$

where γ and Ω are real, positive constants.

(a) Show that

$$y(t) = Ae^{-(\gamma-a)t} + Be^{-(\gamma+a)t} \qquad \text{with} \qquad a = \sqrt{\gamma^2 - \Omega^2} \tag{3}$$

is a general solution to Eq. (2).

(b) Eq. (2). is a second order, ordinary differential equation, implying that we have to specify two initial conditions. Let $y_0 = y(0)$ and $v_0 = dy/dt|_{t=0}$ and express the constants A and B in terms of y_0 and v_0.

Please don't forget to do the PreClass assignment on the Web!