Key Point 2

$\delta^{in}(v)$ = the set of in-edges at node v.

For each $v \neq r$, choose a cheapest edge e_v from $\delta^{in}(v)$.
If $F^* = \{e_v \mid v \in V - \{r\}\}$ is an arborescence, then F^* is optimal. Otherwise, F^* contains a cycle C.

For $(u,v) \in \delta^{in}(v)$, define $c'(u,v) = c(e_v)$
and $c''(u,v) = c(u,v) - c'(u,v)$.
Then, an arborescence F is minimum cost w.r.t c
iff it is miniman cost w.r.t. c''.

Why?

For any arborescence F,

$$c'(F) = \sum_{v \in V - \{r\}} c(e_v).$$

This means that any arborescence is an optimal solution for cost function c'.

Key Point 3

Let C be a cycle not containing r with $c''(C) = 0$.

Then there exists a minimum - cost arborescence

$T = (V, F)$ which enters C exactly once.