Math 2250 Maple Project 2, March 2004. Tacoma Narrows.

NAME ______________________ CLASSTIME ____ VERSION A-E, F-K, L-R, S-Z

Circle the version - see problem 2.1. There are six (6) problems in this project. Please answer the questions A, B, C , ... associated with each problem. The original worksheet "project2-fall-2003.mws" is a template for the solution; you must fill in the code and all comments. Sample code can be copied with the mouse. Use pencil freely to annotate the worksheet and to clarify the code and figures.

The problem headers for the Spring 2004 revision of David Eyre's project (original was year 2000).

__________ 2.1. OVER-DAMPED FREE OSCILLATIONS.
__________2.2. CRITICALLY DAMPED FREE OSCILLATIONS.
__________2.3. UNDER-DAMPED FREE OSCILLATIONS.
__________2.4. UNDAMPED FORCED OSCILLATIONS.
__________2.5. PRACTICAL RESONANCE.
__________2.6. MCKENNA NON-HOOKES LAW CABLE MODEL.

2.1. PROBLEM (OVER-DAMPED FREE OSCILLATIONS)

FREE OSCILLATIONS. Consider the general problem of free linear oscillations

```
m x'' + c x' + k x=0,
x(0)=x0,  x'(0)=v0.
```

Assume m, c and k are non-negative constants. The symbols x0 and v0 are the initial position and initial velocity, respectively. Treated here is the over-damped case c^2 > 4km, page 317 of E&P. Depending on the first letter of your last name, assume:

```
 Version A-E: m=3, k=20       Version F-K: m=4, k=24
 Version L-R: m=3, k=24       Version S-Z: m=5, k=20
```

A. Suggest a value for parameter c > 0 so that the free oscillations are over-damped. This value will be used in item B below. Check your answer by solving the characteristic equation using Maple's "solve" command.

B. Use x(0)=1 and x'(0)= -4 for the initial conditions and Maple's "dsolve" to find the explicit real solution x(t). Plot the solution x(t) for t=0 to t=5 using Maple's "plot" command. Check the plot against Figure 5.4.6 page 317.

EXAMPLE(Wrong parameters! Change it!)

```
# Use semicolons to see what you have done.
de:=3*diff(x(t),t,t)+1.5*diff(x(t),t)
    +4*x(t)=0:                          # Define the differential equation
solve(3*r^2+1.5*r+4=0,r);               # Solve characteristic equation.
ic:=x(0)=0,D(x)(0)= -1:                 # Define the initial conditions
p:=dsolve({de,ic},x(t),method=laplace):# Symbolically solve for x(t)
X:=unapply(rhs(p),t):                   # Capture the dsolve symbolic
                                        # answer as a function X(t)
```

```
plot(X(t),t=0..5);                     # Plot the solution
>
> #2.1-A
> # over-damped means mr^2+cr+k=0 has two real roots.
> #2.1-B
>
2.2 PROBLEM (CRITICALLY DAMPED FREE OSCILLATIONS)

FREE OSCILLATIONS. Consider the free linear oscillation problem

          m x'' + c x' + k x=0,
          x(0)=0,  x'(0)=1.

Here, m, c and k are non-negative constants. The critically damped case
is studied here, c^2 = 4km, as on page 318 in E&P. Depending on the
first letter of your last name, assume:

 Version A-E: m=1, c=5              Version F-K: m=2, c=6
 Version L-R: m=3, c=7              Version S-Z: m=4, c=8

  A. Display the Hooke's constant k > 0 so that the equation is
     critically damped and the solution x(t) changes sign at most once.
     Display the exact symbolic solution x(t), using maple methods from
     the 2.1 example.

  B. Plot the exact symbolic solution x(t) on a suitable t-interval.
     Check the graphic against Figure 5.4.7 page 317 of E&P.


>
> #2.2-A Define  k, then solve.
> # critically damped means mr^2+cr+k=0 has two equal roots.
> #2.2-B Plot.
>
2.3. PROBLEM (UNDER-DAMPED FREE OSCILLATIONS)

FREE OSCILLATIONS. Consider the problem of free linear oscillations

          m x'' + c x' + k x=0,
          x(0)=0,  x'(0)=1.

Here, m, c and k are non-negative constants. The under-damped case
is studied here, c^2 < 4km, as on page 318 in E&P. Depending on
the first letter of your last name, assume:

 Version A-E: m=1, c=3             Version F-K: m=2, c=4
 Version L-R: m=3, c=5             Version S-Z: m=4, c=6

    A. Display a Hooke's constant k > 0 so that the solution x(t) is
       under-damped. Check that x(t)=0 infinitely many times on t>0.
       Display the exact solution x(t) obtained by maple methods
       as in the 2.1 example.

    B. Plot the exact symbolic solution x(t) on a suitable t-interval.
       Check the graphic against Figure 5.4.8 page 318 of E&P.
```

C. Estimate from the graph the decimal value of the pseudo-period. Display the graphical estimate and also the exact pseudo-period 2Pi/w, where w is the natural frequency of the trigonometric term in the solution x(t) found in item 2.3.A.

Maple tip: Click with the mouse on the graphic to print the cursor location (left upper corner of the maple window). The coordinates printed are of the form (x,y). From this coordinate information, a simple subtraction estimates the period

```
>
> #2.3-A Define  k, then solve.
> # under-damped means mr^2+cr+k=0 has two conjugate complex roots.
> #2.3-B Plot.
> #2.3-C Pseudo-period calculations.
>
```

2.4. PROBLEM (UNDAMPED FORCED OSCILLATIONS)

FORCED LINEAR OSCILLATIONS. Consider the undamped (c=0) forced problem

```
mx'' +  k x = 5 cos(wt),
x(0)=0,  x'(0)=0,
```

where m, k and w are non-negative constants. Depending on the first letter of your last name, assume:

Version A-E: m=1, k=2.5   Version F-K: m=2, k=3.5
Version L-R: m=3, k=4.5   Version S-Z: m=4, k=5.5

A. Choose the forcing angular frequency w to be 3 times larger than the natural angular frequency w0, w0^2=k/m. Solve for x(t) using dsolve(). Plot the solution x(t) on a suitable interval in order to show the global behavior of the solution x(t). See Figure 5.6.2, page 341.

B. The solution x(t) is the sum of two functions, one of period 2Pi/w and the other of period 2Pi/w0. Display the exact period, as calculated from the solution formula for x(t) -- see page 341 for details.

C. Suggest a value for the forcing frequency w so that the oscillations exhibit resonance. Show resonant behavior on a graph. Check against Figure 5.6.4, page 343.

```
> #2.4-A
> #2.4-B
> #2.4-C
>
```

2.5. PROBLEM (PRACTICAL RESONANCE)

Consider the damped forced problem

```
mx'' + c x' + k x = 5 cos(w t),
x(0)=0,  x'(0)=0.
```