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BMGT230 Midterm 2

Chapter 8: Random Variables & Probability Models

- **expected value-** $\mu = EV = E(X) = \Sigma x \cdot P(x)$
  - $E(X+Y) = E(X)+E(Y)$
  - $E(X-Y) = E(X)-E(Y)$
  - $E(2X)=2E(X)$
  - $E(X+C)=E(X)+C$
- **variance-** $\sigma^2 = Var(X) = \Sigma(x-\mu)^2 P(x)$
  - $Var(x)=E[(x-\mu)^2]=E(x^2)-\mu^2$
  - $Var(X+Y)=Var(X) + Var(Y)$
  - $Var(X-Y)= Var(X) + Var(Y)$
  - $Var(X+C)=Var(X)$
  - $Var(a+bx)=b^2 var(x)$

Assumptions
$np>10$ $nq>10$
random
independent
10% of population

- **Standard deviation-** $\sigma = SD(x)=\sqrt{var(x)}= \sqrt{\Sigma p(x)*(x-\mu)^2}$
- **Z score-** $(x-\mu) / \sigma$

$z = \frac{\text{value - mean}}{SD}$

Normal Distribution: $N(\mu,\sigma)$
- $\mu= E(X)=np$
- $\sigma^2=Var(x)=npq$
- $\sigma = \sqrt{npq}$

Binomial Distribution: $B(n,p)$
- $P(x=k)= [n! / x!(n-x)!][p^x q^{n-x}]$ x=successes; n=trials; p=prob of success; q=prob of failure
- $z=(x-np) / \sqrt{npq}$

use binomial program
or
binompdf (n, p, x): spec number of successes
binomcdf (n, p, x): values $\leq x$

Chapter 9: Sampling Distributions and Confidence Intervals for Proportions

Sample Proportions:
- $\hat{p} = x/n$ x=successes
- $\sigma = \sqrt{\hat{p}\hat{q}/n}$ *use p=1/2 when not given
- $\hat{p} \sim N(p,\sqrt{pq/n})$
- $z=(\hat{p}-p_0)/ (\sqrt{pq/n})$

Confidence Intervals for Proportions: $\hat{p} \pm z^*\sqrt{\hat{p}\hat{q}/n}$
Margin of Error pop. prop. is: $z^*\sqrt{\hat{p}\hat{q}/n}$ z* depends on CI%

$SD(\hat{p}) = \sqrt{\frac{pq}{n}}$
$SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}}$

SE is the estimate of SD

Z* Critical Values:

CL	A	1-side	2-side
90%	.1	1.28	1.645
95%	.05	1.645	1.96
99%	.01	2.33	2.576
98%			2.492

Sample Size:
- $p \sim N(p,\sqrt{pq/n})$
- $n=[z^{*2}pq]/[m^2]$ *if p is unknown, use p=1/2

Chapter 10: Testing Hypotheses for Proportions

One Proportion Z-Test:

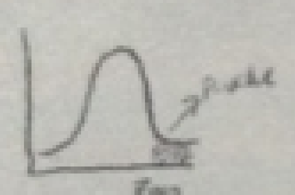
1. Null, Alternative

H_0: $p=p_0$
H_A: $p<p_0$, $p>p_0$ — one-sided; $p \neq p_0$ — two-sided

2. Test Statistic (z)

$z_{obs} = \frac{\hat{p}-p_0}{SD(\hat{p})} \Rightarrow \frac{\hat{p}-p_0}{\sqrt{\frac{p_0 q_0}{n}}}$
$SD(\hat{p}) = \sqrt{\frac{p_0 q_0}{n}}$

3. P-value



normalcdf $(z,\infty) = P$
$(-\infty,-z) = P$
$(-z,z) = P$

4. P vs. Alpha
$P<\alpha$, reject $P>\alpha$, fail to reject

Rejection Regions:
1. $p<p_0$, reject is $z<-z_\alpha$
2. $p>p_0$, reject if $z>z_\alpha$
3. $p \neq p_0$, reject is z falls in region

reject H_0 if z_{obs} falls into a z_α region

Confidence Intervals: reject is value is outside interval; fail to reject if value is inside interval: $CI= \hat{p} \pm z^*\sqrt{\hat{p}\hat{q}/n}$ Z interval

Chapter 11: Hypotheses Tests for Means

σ is given- use z $z=(x-\bar{x}) / \sigma$
σ is not given- use t $t= [\bar{x}-\mu] / [s\sqrt{n}]$

$x \sim N(\mu,\sigma/\sqrt{n})$
df = n-1

$SE(x)=s\sqrt{n}$

$CI=\bar{x} \pm z^*[s/\sqrt{n}]$
$CI: \bar{x} \pm t^*\left(\frac{s}{\sqrt{n}}\right)$

One Sample T-Test:

1. H_0: $\mu_0=\mu$
H_A: $\underbrace{\mu_0<\mu \quad \mu_0>\mu}_{\text{one sided}}$, $\underbrace{\mu_0 \neq \mu}_{\text{2 sided}}$

2. $t= [x-\mu] / [s\sqrt{n}]$

3. P-value

4. $P<\alpha$, reject $P>\alpha$, fail to reject

Rejection Region:
1. $\mu<\mu_0$, reject is $t<-t_\alpha$
2. $\mu>\mu_0$, reject if $t>t_\alpha$
3. $\mu \neq \mu_0$, reject is t falls in region

Confidence Interval:
$\bar{x} \pm t [s\sqrt{n}]$

Sample Size:
Case I: σ is known $n= [z^2\sigma^2] / [m^2]$

Case II: σ is unknown $n_1= [z^2s^2] / [m^2]$ use n_1 for df to find t*
$n_2= [t^2s^2] / [m^2]$ *always round up

Type I Error: reject H_0 when it is true – α
Type II Error: fail to reject H_0 when H_A is true – β

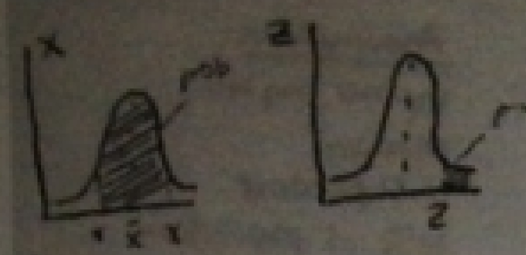
smaller p-value → strong evidence against H_0

CI: "we are 95% confident that the true proportion is in the interval"

Calc. Functions

normalcdf (lower x, upper x, mean, SD) = probability [2nd][Vars]

normalcdf (lower z, upper z) = p-value



binompdf (n, p, x) find # of successes [2nd][

Ex. flip a coin 10 times, prob of 8 heads?

binomcdf (n, p, x) find values ≤ x [2nd][

Ex. Roll dice 10 times, prob less than 3 5's

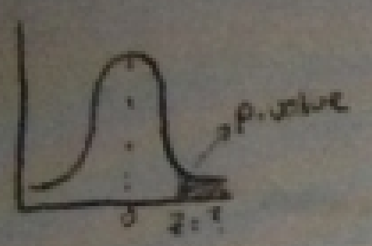
binompdf (10, 1/6, 3)

⇓ conti

invNorm (p-value) = z-value [2nd][Vars]

use when given percentile to find x

$z = \frac{x - \bar{x}}{\sigma}$



invT(area, df) = t_{n-1} [2nd][Vars]

Ex. CI = 95%, n=11

invT(.975, 10)

two-sided

one-sided:

invT (.95, 10)

Ex. Prob = 25% n=10

invT(.25, 9)

Z_{test} [Stat] → Tests

- input μ_0, $\bar{x}$, σ, n, claim
- gives you p-val and z-val
- compare p to α

$Z_{interval}$ (when σ is known) [Stat] → Tests

- input σ, $\bar{x}$, n, CI
- output is interval

T_{test} [Stat] → Tests

- input μ_0, $\bar{x}$, s, n, claim
- gives you t-val and p-val
- compare p to α

$T_{interval}$ (when σ is unknown) [Stat] → Tests

- input $\bar{x}$, s, n, CI
- output is interval

Binomial Probability

1. only 2 outcomes
2. n: trials
   q: prob of failure
   p: prob of success

To find $P(X = \#)$

→ binomial pdf (n, p, x)

To find $P(X < \#)$

→ binomial cdf (n, p, x)

To find $P(X > \#) \Rightarrow 1 - P(X < \# - 1)$

→ binomial cdf (n, p, x-1)

Ex. prob at least 7 heads?

$P(x \geq 7) = 1 - P(x \leq 6)$

binomcdf (10, .5, 6) = 17%

p: true population proportion

p_0: hypothesized value

$\hat{p}$: observed proportion