

1. Let $\Sigma = \{a,b\}$. Define in set notation the following languages

a. $\{w \in \Sigma^* \mid |w| \equiv 1 \pmod{3}\}$

b. $\{w \in \Sigma^* \mid |w| \equiv 0 \pmod{3}\}$

c. $\{w \in \Sigma^* \mid w \in \{aa,ab,ba\}^*\}$

d. $\overline{\{w \in \Sigma^* \mid |w| \equiv 1 \pmod{3}\}}$

2. a. Argue as to why it is possible to design an FA to accept the following language

$$L = \{ w \in \{a,b,c\}^* \mid (n_a(w) + n_b(w)) \pmod{3} = n_c(w) \pmod{4} \}.$$

b. How many states would this dfa have? Support your answer.

3. For each of the following languages, prove that it is a regular language by construction or cogent argument or prove that it is not, by careful application of the pumping lemma. You may also use any results we obtained in class.

a. $L = \{ a^k b^l c^m \mid k+l+m < n, n \in \mathbb{N} \}$

b. $L = \{ a^n b^n c^n d^n \mid n \geq 0 \}$

4. For the following languages, prove whether they are context free. Assume $\Sigma = \{a, b\}$.

a. $L_1 = \{ a^n b^m \mid n \geq 2m \}$

b. $L_2 = \{ ww^r ww^r \mid w \in \Sigma^+ \}$

5. Consider the following CFG. Transform this grammar into Chomsky Normal Form. Be sure to remove lambda and unit productions before proceeding.

$S \rightarrow abAB$
 $A \rightarrow bAB \mid \lambda$
 $B \rightarrow BAa \mid A \mid \lambda$

6. Describe the strategy for building a Turing machine to accept the following language

$$L = \{ w = a^i \mid i = 2^k, \text{ where } k \text{ is a nonnegative integer} \}$$