

Math 2250 Maple Project 2, October 2003. Tacoma Narrows.

NAME _____ CLASSTIME ____ VERSION A-E, F-K, L-R, S-Z

Circle the version - see problem 2.1. There are six (6) problems in this project. Please answer the questions A, B, C, ... associated with each problem. The original worksheet "project2-fall-2003.mws" is a template for the solution; you must fill in the code and all comments. Sample code can be copied with the mouse. Use pencil freely to annotate the worksheet and to clarify the code and figures.

The problem headers for the Fall 2003 revision of David Eyre's project (original was year 2000).

- _____ 2.1. OVERDAMPED FREE OSCILLATIONS.
- _____ 2.2. CRITICALLY DAMPED FREE OSCILLATIONS.
- _____ 2.3. UNDERDAMPED FREE OSCILLATIONS.
- _____ 2.4. UNDAMPED FORCED OSCILLATIONS.
- _____ 2.5. PRACTICAL RESONANCE.
- _____ 2.6. MCKENNA NON-HOOKES LAW CABLE MODEL.

2.1. PROBLEM (OVERDAMPED FREE OSCILLATIONS)

FREE OSCILLATIONS. Consider the general problem of free linear oscillations

$$m x'' + c x' + k x = 0,$$
$$x(0) = x_0, \quad x'(0) = v_0.$$

Assume m , c and k are non-negative constants. The symbols x_0 and v_0 are the initial position and initial velocity, respectively. Treated here is the overdamped case $c^2 > 4km$, page 317 of E&P. Depending on the first letter of your last name, assume:

Version A-E: $m=1$, $k=16$ Version F-K: $m=2$, $k=20$
Version L-R: $m=3$, $k=18$ Version S-Z: $m=4$, $k=24$

- A. Suggest a value for parameter $c > 0$ so that the free oscillations are overdamped. This value will be used in item B below. Check your answer by solving the characteristic equation using Maple's "solve" command.

- B. Use $x(0)=1$ and $x'(0) = -2$ for the initial conditions and Maple's "dsolve" to find the explicit real solution $x(t)$. Plot the solution $x(t)$ for $t=0$ to $t=5$ using Maple's "plot" command. Check the plot against Figure 5.4.6 page 317.

EXAMPLE(Wrong parameters! Change it!)

Use semicolons to see what you have done.

```
de:=3*diff(x(t),t,t)+1.5*diff(x(t),t)
+4*x(t)=0: # Define the differential equation
solve(3*r^2+1.5*r+4=0,r); # Solve characteristic equation.
ic:=x(0)=0,D(x)(0)= -1: # Define the initial conditions
p:=dsolve({de,ic},x(t),method=laplace): # Symbolically solve for x(t)
X:=unapply(rhs(p),t): # Capture the dsolve symbolic
# answer as a function X(t)
```

```

plot(X(t),t=0..5);          # Plot the solution
>
> #2.1-A
> # overdamped means  $mr^2+cr+k=0$  has two real roots.
> #2.1-B
>
2.2 PROBLEM (CRITICALLY DAMPED FREE OSCILLATIONS)

```

FREE OSCILLATIONS. Consider the free linear oscillation problem

$$m x'' + c x' + k x = 0, \\ x(0) = 0, \quad x'(0) = 1.$$

Here, m , c and k are non-negative constants. The critically damped case is studied here, $c^2 = 4km$, as on page 318 in E&P. Depending on the first letter of your last name, assume:

Version A-E: $m=1, c=4$ Version F-K: $m=2, c=5$
Version L-R: $m=3, c=7$ Version S-Z: $m=4, c=6$

- A. Display the Hooke's constant $k > 0$ so that the equation is critically damped and the solution $x(t)$ changes sign at most once. Display the exact symbolic solution $x(t)$, using maple methods from the 2.1 example.
- B. Plot the exact symbolic solution $x(t)$ on a suitable t -interval. Check the graphic against Figure 5.4.7 page 317 of E&P.

```

>
> #2.2-A Define k, then solve.
> # critically damped means  $mr^2+cr+k=0$  has two equal roots.
> #2.2-B Plot.
>
2.3. PROBLEM (UNDERDAMPED FREE OSCILLATIONS)

```

FREE OSCILLATIONS. Consider the problem of free linear oscillations

$$m x'' + c x' + k x = 0, \\ x(0) = 0, \quad x'(0) = 1.$$

Here, m , c and k are non-negative constants. The underdamped case is studied here, $c^2 < 4km$, as on page 318 in E&P. Depending on the first letter of your last name, assume:

Version A-E: $m=1, c=4$ Version F-K: $m=2, c=5$
Version L-R: $m=3, c=7$ Version S-Z: $m=4, c=6$

- A. Display the Hooke's constant $k > 0$ so that the solution $x(t)$ is underdamped and $x(t)$ changes sign infinitely often on $t > 0$. Display the exact solution $x(t)$ obtained by maple methods from the 2.1 example.
- B. Plot the exact symbolic solution $x(t)$ on a suitable t -interval. Check the graphic against Figure 5.4.8 page 318 of E&P.

- C. Estimate from the graph the decimal value of the pseudoperiod. Display the graphical estimate and also the exact pseudoperiod $2\pi/\omega$, where ω is the natural frequency of the trigonometric term in the solution $x(t)$ found in item 2.3.A.

Maple tip: Click with the mouse on the graphic to print the cursor location (left upper corner of the maple window). The coordinates printed are of the form (x,y) . From this coordinate information, a simple subtraction estimates the period

```
>
> #2.3-A Define k, then solve.
> # underdamped means  $m r^2 + cr + k = 0$  has two conjugate complex roots.
> #2.3-B Plot.
> #2.3-C Pseudoperiod calculations.
>
```

2.4. PROBLEM (UNDAMPED FORCED OSCILLATIONS)

FORCED LINEAR OSCILLATIONS. Consider the undamped ($c=0$) forced problem

$$m x'' + k x = 10 \cos(\omega t),$$

$$x(0)=0, \quad x'(0)=0,$$

where m , k and ω are non-negative constants. Depending on the first letter of your last name, assume:

Version A-E: $m=1, k=1.5$	Version F-K: $m=2, k=2.5$
Version L-R: $m=3, k=3.5$	Version S-Z: $m=4, k=4.5$

- A. Choose the forcing angular frequency ω to be 3 times larger than the natural angular frequency ω_0 , $\omega_0^2=k/m$. Solve for $x(t)$ using `dsolve()`. Plot the solution $x(t)$ on a suitable interval in order to show the global behavior of the solution $x(t)$. See Figure 5.6.2, page 341.
- B. The solution $x(t)$ is the sum of two functions, one of period $2\pi/\omega$ and the other of period $2\pi/\omega_0$. Display the exact period, as calculated from the solution formula for $x(t)$ -- see page 341 for details.
- C. Suggest a value for the forcing frequency ω so that the oscillations exhibit resonance. Show resonant behavior on a graph. Check against Figure 5.6.4, page 343.

```
> #2.4-A
> #2.4-B
> #2.4-C
>
```

2.5. PROBLEM (PRACTICAL RESONANCE)

Consider the damped forced problem

$$m x'' + c x' + k x = 10 \cos(\omega t),$$

$$x(0)=0, \quad x'(0)=0.$$