



Relativity 2

Disclaimer: These lecture notes are not meant to replace the course textbook. The content may be incomplete or even inaccurate. Some topics may be unclear. These notes are only meant to be a study aid and a supplement to your own notes. Please report any inaccuracies to the professor.

Einstein's Postulates

The absence of any fringe shift in the Michelson-Morely experiment for any orientation of the interferometer and for any time of the year negated the ether hypothesis for light propagation. Light waves are oscillations of the electromagnetic field, and no propagation medium is necessary, unlike sound waves. However, if Galilean transformations are correct, then Maxwell's equations must be modified for every possible reference frame to account for different velocities for the speed of light. Einstein assumed the opposite: that Maxwell's equations are fundamentally correct, but that our intuitive Galilean transformation is not. This led to the following two postulates:

1. The laws of physics, including electromagnetism, are the same in all inertial frames.
2. Every observer measures the same value c for the speed of light (in vacuum) in all inertial frames.

The second postulate is really a consequence of the first, because if Maxwell's equations hold in all inertial frames, then the only possible value for the speed of light is c . These postulates embody Einstein's **Special Theory of Relativity**, first published in 1905 in a paper titled On the Electrodynamics of Moving Bodies. Later he would incorporate gravity and acceleration in his General Theory of Relativity. As in Newtonian Relativity, there is no way to detect absolute motion. Only the relative velocities between two inertial reference frames matters.

These seemingly simple postulates have extraordinary consequences. For example, when you turn on the headlights of a car, the light beam leaves the car at a relative velocity of $c = 3.0 \times 10^8$ m/s. However, someone standing on the sidewalk also measures the speed of the light beam as c independent of the velocity of the car! How can this be? As we shall see, our concepts of space and time must be modified.

Basic Definitions

Events are physical phenomena that occur independent of any reference frame. For example: a flash, explosion, return of a spaceship, or disintegration of a subatomic particle.

Observers record events, both the time and spatial coordinates, in a particular reference frame. For example, Mission Control in Houston marking down the time and location of the splashdown of a space capsule. The reference frame in this case is the Earth.

Simultaneous events occur when the light signals from two events reach an observer at the same time

Relativity of Simultaneity:

Two events simultaneous in one inertial frame are not simultaneous in any other frame. This is a consequence of Einstein's Postulates.

Proper time is the time difference between two events occurring at the same position (Denoted by t_0 or τ).

Rest frame is the inertial frame where two events are only separated by time. The frame in which the proper time is measured

Proper length is the distance between two positions at rest, the length measured in the rest frame. (Denoted by L_0).

Now that we are armed with these definitions, let's explore the consequences of the constancy of the speed of light in all inertial frames.

Time Dilation

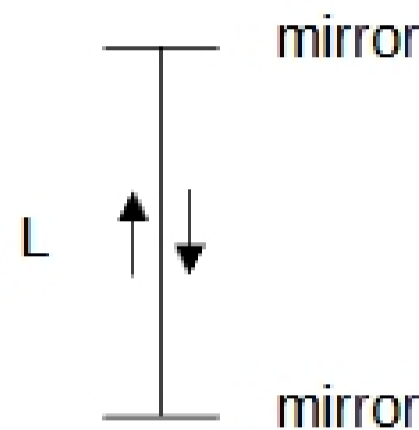
We explore the rate of time in different inertial frames by considering a special kind of clock – a light clock – which is just one arm of an interferometer. Consider a light pulse bouncing vertically between two mirrors. We analyze the time it takes for the light pulse to complete a round trip both in the rest frame of the clock (labeled S'), and in an inertial frame where the clock is observed to move horizontally at a velocity v (labeled S).

In the rest frame S'

$$t_1' = \frac{L}{c} = \text{time up}$$

$$t_2' = \frac{L}{c} = \text{time down}$$

$$\tau = t_1' + t_2' = \frac{2L}{c}$$



Now put the light clock on a spaceship, but measure the roundtrip time of the light pulse from the Earth frame S :

$$t_1 = \frac{t}{2} = \text{time up}$$

$$t_2 = \frac{t}{2} = \text{time down}$$

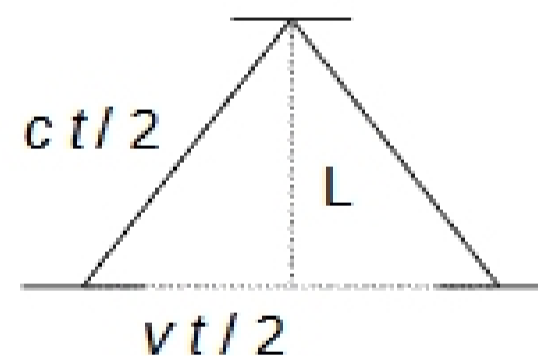
The speed of light is still c in this frame, so

$$L^2 + v^2 t^2 / 4 = c^2 t^2 / 4$$

$$L^2 = c^2 t^2 / 4 - v^2 t^2 / 4$$

$$t^2 = \frac{4L^2}{c^2 - v^2}$$

$$t = \frac{2L}{c} \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{\tau}{\sqrt{1 - v^2/c^2}}$$



So the time it takes the light pulse to make a roundtrip in the clock when it is moving by us appears longer than when it is at rest. We say that time is **dilated**. It also doesn't matter which frame is the Earth and which is the clock. Any object that moves by with a significant velocity appears to have a clock running slow. We summarize this effect in the following relation:

$$t = \gamma \tau \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \geq 1$$