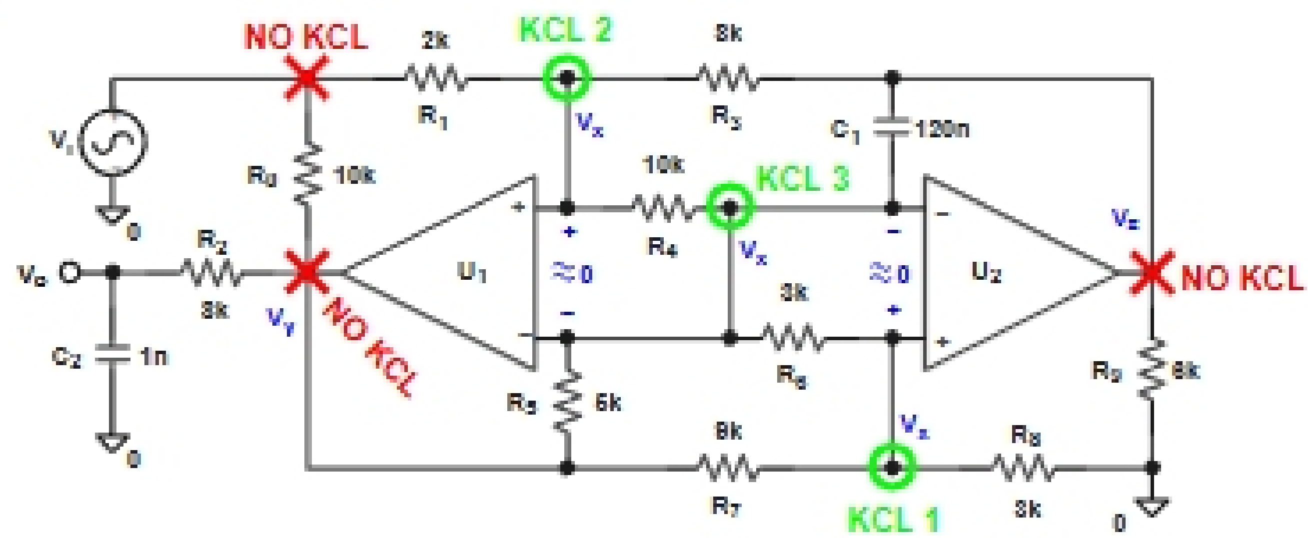
OPERATIONAL AMPLIFIERS

Solution - Dr. Aydın İ. Karşılayan

1.(a)



$$\text{KCL 1} \;\Rightarrow\; \cancel{\frac{v_x - v_x}{R_6}} + \frac{v_x - v_y}{R_7} + \frac{v_x}{R_8} = 0 \Rightarrow (R_7 + R_8)v_x = R_8 v_y \Rightarrow 12v_x = 3v_y \Rightarrow \boxed{v_x = \frac{v_y}{4}}$$

$$\text{KCL 2} \;\Rightarrow\; \frac{v_x - v_i}{R_1} + \frac{v_x - v_z}{R_3} + \cancel{\frac{v_x - v_x}{R_4}} = 0 \Rightarrow (R_1 + R_3)v_x = R_3 v_i + R_1 v_z \Rightarrow 10v_x = 8v_i + 2v_z$$

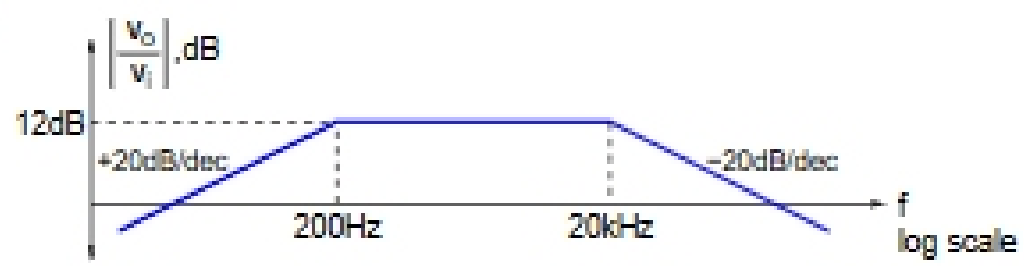
$$\Rightarrow\; v_z = 5v_x - 4v_i \Rightarrow \boxed{v_z = \frac{5}{4}v_y - 4v_i}$$

$$\text{KCL 3} \;\Rightarrow\; \cancel{\frac{v_x - v_x}{R_4}} + \cancel{\frac{v_x - v_x}{R_6}} + sC_1(v_x - v_z) + \frac{v_x - v_y}{R_5} = 0 \Rightarrow sC_1\left(\frac{v_y}{4} - \left(\frac{5}{4}v_y - 4v_i\right)\right) + \frac{\frac{v_y}{4} - v_y}{R_5} = 0$$

$$\Rightarrow\; \left(sC_1 + \frac{3}{4R_5}\right)v_y = 4sC_1 v_i \Rightarrow \frac{v_y}{v_i} = \frac{4sC_1}{sC_1 + \frac{3}{4R_5}} = \frac{4s}{s + \frac{3}{4R_5C_1}} \Rightarrow \boxed{\frac{v_y}{v_i} = \frac{4s}{s + 2\pi 200}}$$

$$\frac{v_o}{v_y} = \frac{\frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = \frac{1}{1 + sR_2C_2} \Rightarrow \boxed{\frac{v_o}{v_y} = \frac{1}{1 + \frac{s}{2\pi 20k}}} \Rightarrow \frac{v_o}{v_i} = \frac{v_y}{v_i}\frac{v_o}{v_y} = \boxed{4\,\frac{s}{s + 2\pi 200}\;\frac{1}{1 + \frac{s}{2\pi 20k}}}$$

(b)



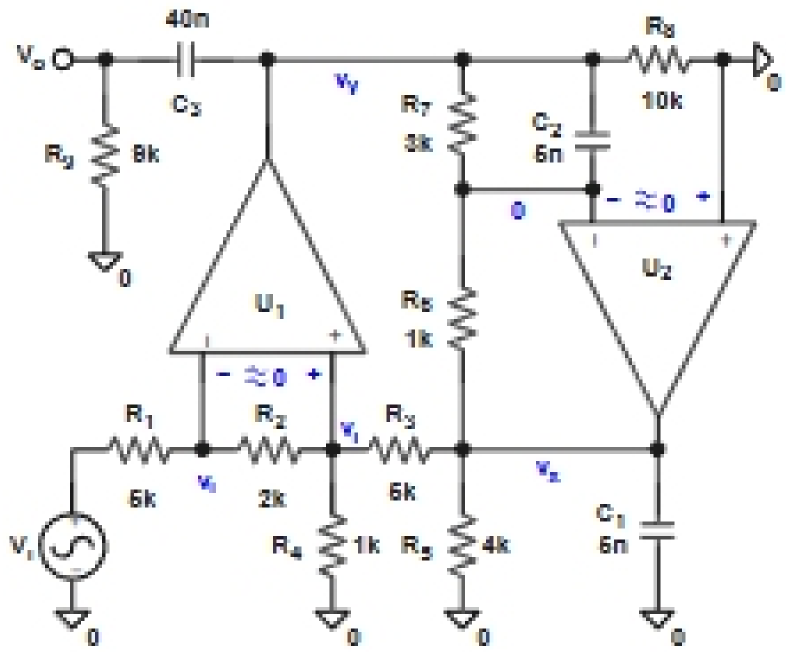
(c) $v_i(t) = 0.1\sin(2\pi 10^3 t)$

$$\left|\frac{v_o}{v_i}(j2\pi 10^3)\right| = 4\,\frac{1}{\sqrt{1 + \left(\frac{200}{1000}\right)^2}}\;\frac{1}{\sqrt{1 + \left(\frac{1}{20}\right)^2}} = 3.9$$

$$\angle\frac{v_o}{v_i}(j2\pi 10^3) = \tan^{-1}\frac{200}{1000} - \tan^{-1}\frac{1}{20} = 8.4^\circ$$

$$v_o(t) = 3.9 \times 0.1\sin\left(2\pi 10^3 t + \frac{8.4}{180}\pi\right) = 0.39\sin(2\pi 10^3 t + 0.15)$$

2.(a)



$$i_{R2} = \frac{v_{d1}}{R_2} = 0 \Rightarrow i_{R1} = i_{R2} = 0 \Rightarrow i_{R3} = i_{R4}$$

$\Rightarrow R_3$ and R_4 form a voltage divider

$$\Rightarrow \frac{v_i}{v_x} = \frac{R_4}{R_3 + R_4} = \frac{1k}{5k + 1k} = \frac{1}{6} \Rightarrow \boxed{\frac{v_x}{v_i} = 6}$$

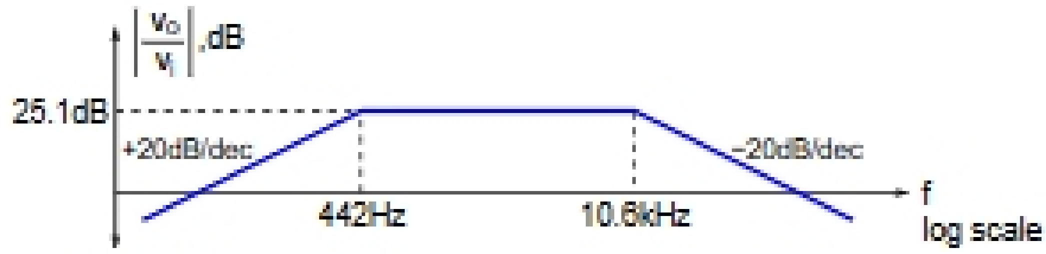
$$\frac{v_x}{v_y} = \frac{-R_6}{\frac{1}{sC_2} \parallel R_7} = \frac{-R_6}{\frac{R_7}{1 + sR_7C_2}} = \frac{-R_6}{R_7}(1 + sR_7C_2)$$

$$\Rightarrow \frac{v_y}{v_x} = \frac{-\frac{R_7}{R_6}}{1 + sR_7C_2} = \boxed{\frac{-3}{1 + \frac{s}{2\pi 10.6k}}}$$

$$\frac{v_o}{v_y} = \frac{R_9}{\frac{1}{sC_3} + R_9} = \frac{s}{s + \frac{1}{R_9C_3}} = \boxed{\frac{s}{s + 2\pi 442}}$$

$$\Rightarrow \frac{v_o}{v_i} = \frac{v_x}{v_i}\frac{v_y}{v_x}\frac{v_o}{v_y} = \boxed{-18\,\frac{s}{s + 2\pi 442}\,\frac{1}{1 + \frac{s}{2\pi 10.6k}}}$$

(b)



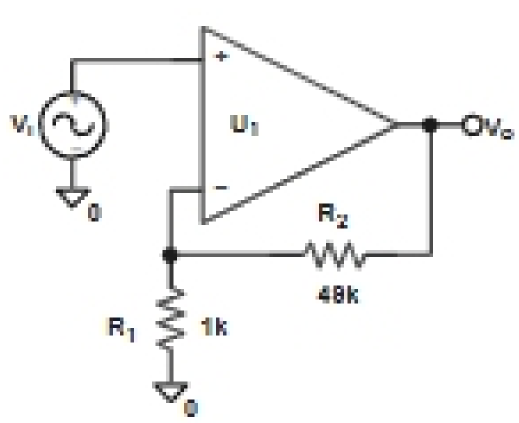
(c) $v_i(t) = 0.1\sin(2\pi 10^3 t)$

$$\left|\frac{v_o}{v_i}(j2\pi 10^3)\right| = 18\,\frac{1}{\sqrt{1 + \left(\frac{442}{1000}\right)^2}}\,\frac{1}{\sqrt{1 + \left(\frac{1}{10.6}\right)^2}} = 16.4$$

$$\angle\frac{v_o}{v_i}(j2\pi 10^3) = -180^o + \tan^{-1}\frac{442}{1000} - \tan^{-1}\frac{1}{10.6} = -161.5^o$$

$$v_o(t) = 16.4 \times 0.1\sin\left(2\pi 10^3 t + \frac{-161.5}{180}\pi\right) = 1.64\sin(2\pi 10^3 t - 2.8)$$

3. Using the non-inverting configuration, the amplifier can be designed as follows:



$$\frac{V_o}{V_i} = 1 + \frac{R_2}{R_1} = 1 + \frac{49k}{1k} = 50$$

$f_{3db} = 30kHz \Rightarrow$ Opamp's GBW $= 50 \times 30kHz = \boxed{1.5MHz}$

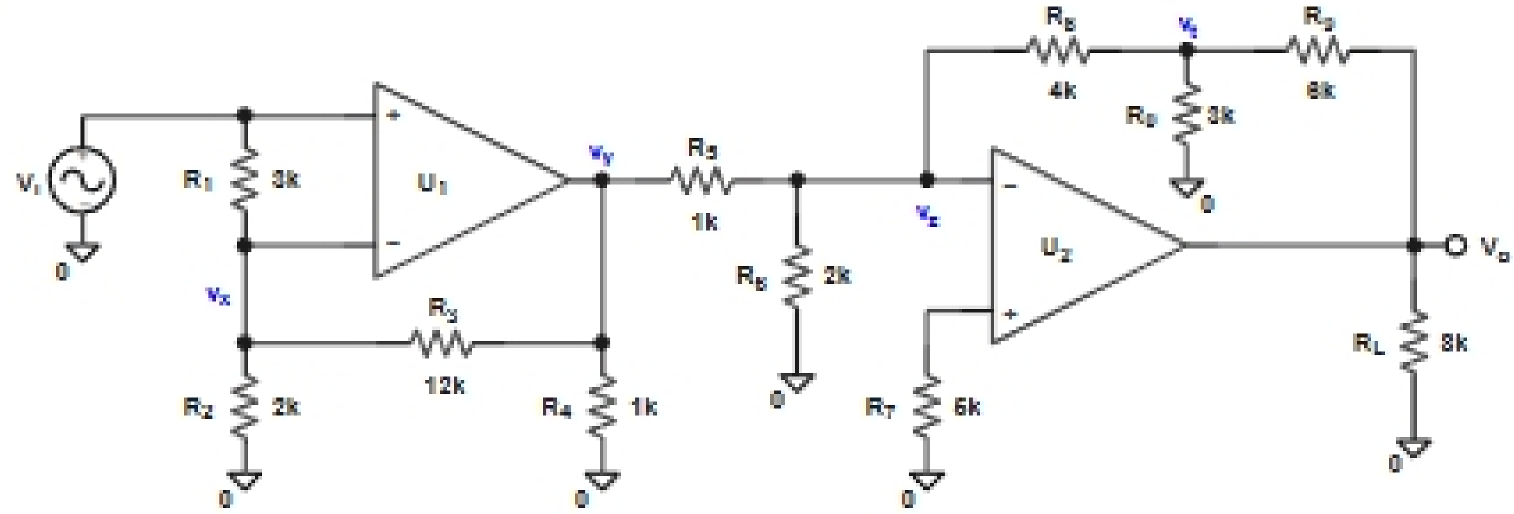
For the maximum rate of change at V_o: $V_i(t) = 0.1\sin(2\pi 30 \times 10^3 t)$

$$\Rightarrow V_o(t) = 0.1\frac{50}{\sqrt{2}}\sin\left(2\pi 30 \times 10^3 t + \frac{\pi}{4}\right)$$

$$\Rightarrow \left.\frac{dV_o(t)}{dt}\right|_{max} = 2\pi 30 \times 10^3 \times 0.1\frac{50}{\sqrt{2}} = 0.67 \times 10^6 V/s = \boxed{0.67V/\mu s}$$

The minimum requirements for the opamp's gain-bandwidth product and slew rate are 1.5 MHz and 0.67 V/μs, respectively.

4.



(a) $\frac{v_x - v_i}{R_1} + \frac{v_x}{R_2} + \frac{v_x - v_y}{R_3} = 0 \Rightarrow \frac{v_x - v_i}{3} + \frac{v_x}{2} + \frac{v_x - v_y}{12} = 0 \Rightarrow 4(v_x - v_i) + 6v_x + v_x - v_y = 0 \Rightarrow \boxed{v_x = \frac{4v_i + v_y}{11}}$

$$v_y = A(s)(v_i - v_x) \Rightarrow v_x = v_i - \frac{v_y}{A(s)} = \frac{4v_i + v_y}{11} \Rightarrow \frac{v_y}{v_i} = \frac{7}{1 + \frac{11}{A(s)}}$$

$$\Rightarrow \frac{v_y}{v_i} = \frac{7}{1 + 11\frac{\left(1 + \frac{s}{2\pi 10}\right)}{10^5}} = \frac{7}{1 + \cancel{11 \times 10^{-5}} + \frac{s}{2\pi 91k}} \approx \boxed{\frac{7}{1 + \frac{s}{2\pi 91k}}}$$

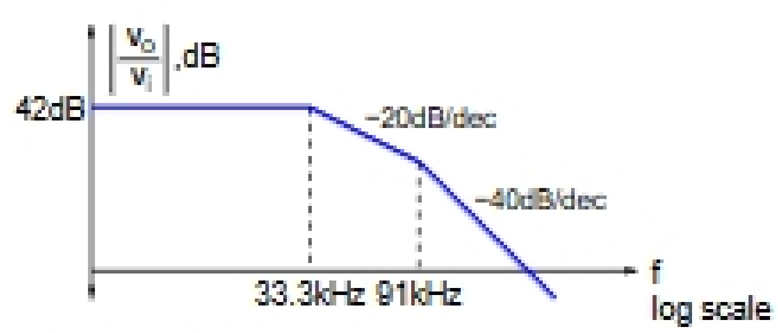
$$\frac{v_t - v_z}{R_8} + \frac{v_t}{R_0} + \frac{v_t - v_o}{R_9} = 0 \Rightarrow \frac{v_t - v_z}{4} + \frac{v_t}{3} + \frac{v_t - v_o}{6} = 0 \Rightarrow 3(v_t - v_z) + 4v_t + 2(v_t - v_o) = 0 \Rightarrow \boxed{v_t = \frac{3v_z + 2v_o}{9}}$$

$$\frac{v_z - v_y}{R_5} + \frac{v_z}{R_6} + \frac{v_z - v_t}{R_8} = 0 \Rightarrow \frac{v_z - v_y}{1} + \frac{v_z}{2} + \frac{v_z - v_t}{4} = 0 \Rightarrow 4(v_z - v_y) + 2v_z + v_z - v_t = 0 \Rightarrow v_z = \frac{4v_y + v_t}{7}$$

$$\Rightarrow v_z = \frac{4}{7}v_y + \frac{1}{7}\left(\frac{3v_z + 2v_o}{9}\right) \Rightarrow v_z = \frac{3}{5}v_y + \frac{1}{30}v_o = \frac{-v_o}{A(s)} \Rightarrow \frac{v_o}{v_y} = \frac{-18}{1 + \frac{30}{A(s)}}$$

$$\Rightarrow \frac{v_o}{v_y} = \frac{-18}{1 + 30\frac{\left(1 + \frac{s}{2\pi 10}\right)}{10^5}} = \frac{-18}{1 + \cancel{30 \times 10^{-5}} + \frac{s}{2\pi 33.3k}} \approx \boxed{\frac{-18}{1 + \frac{s}{2\pi 33.3k}}} \Rightarrow \boxed{\frac{v_o}{v_i} = -126\,\frac{1}{1 + \frac{s}{2\pi 33.3k}}\,\frac{1}{1 + \frac{s}{2\pi 91k}}}$$

(b)



(c) $v_i(t) = 0.01\sin(2\pi 10^4 t)$

$$\left|\frac{v_o}{v_i}(j2\pi 10^4)\right| = 126\,\frac{1}{\sqrt{1 + \left(\frac{10}{33.3}\right)^2}}\,\frac{1}{\sqrt{1 + \left(\frac{10}{91}\right)^2}} = 120$$

$$\angle\frac{v_o}{v_i}(j2\pi 10^4) = -180^\circ - \tan^{-1}\frac{10}{33.3} - \tan^{-1}\frac{10}{91} = -203^\circ$$

$$v_o(t) = 120 \times 0.01\sin\left(2\pi 10^4 t + \frac{-203}{180}\pi\right) = 1.2\sin(2\pi 10^4 t - 3.5)$$

(d) $v_i(t) = A_i\sin(2\pi 25 \times 10^3 t)$

$$\left|\frac{v_o}{v_y}(j2\pi 25 \times 10^3)\right| = 18\,\frac{1}{\sqrt{1 + \left(\frac{25}{33.3}\right)^2}} = 14.4 \Rightarrow |v_o| > |v_y| \Rightarrow v_o \text{ distorts first}$$