Section 2.6 Derivatives and Rates of Change

If an object travels in a straight line at a constant velocity, v, and f(t) is the distance from a reference point at time t, then f(t)-f(a) is the distance traveled from time a to time t. We know f(t)-f(a)=v(t-a) or distance =(rate)( time).
If the velocity was not constant, we talked about the average velocity over a time interval and the instantaneous velocity at a. See section 2.1.

If we think of a graph of a continuous function in terms of distance and time then the x-axis is the time variable, the value of f(x) is the distance. The average velocity is the **average rate of change**, AVROC, of the function and is the slope of the secant line connecting P(a , f(a)) and Q(x, f(x)).

Slope of Secant = $\frac{f(x)-f(a)}{x-a} = \frac{f(a+h)-f(a)}{h}$ *where* $x = a+h$

The instantaneous velocity is the **instantaneous rate of change**, ROC, and is the limit of the slopes of the secant lines as x approaches but is not equal to a, if this limit exists. This is the slope of the tangent line to the graph at a, and is called $f'(a)$, read *"f prime at a"*.

Slope of Tangent $= f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ if this limit exists.

Equation of Tangent line: The tangent line at *(a, f(a))* passes through *(a, f(a))* and has slope *f'(a)* so the equation using pt-slope is

$y - f(a) = f'(a)(x-a)$ *or* $y = f'(a)(x-a) + f(a)$

Examples: 1) f(x) = mx + b is linear. Then any secant line is the line itself and has the same slope, m. $\lim_{h \to 0} m = m$
The derivative of any line is the slope of the line, the derivative of a constant is 0.

2) $f(x) = x^2$ $a = 3$ Find *f'*(3). First write the secant slope for 3+h and 3 and simplify it. Then let h approach 0.

secant slope = $\frac{(3+h)^2-9}{h} = \frac{(9+6h+h^2)-9}{h} = \frac{6h+h^2}{h} = \frac{h(6+h)}{h} = 6+h$

tangent slope = $f'(3) = \lim_{h \to 0}(6+h) = 6$

The rate of change of *f(x)* at 3 is 6. The height is increasing 6 times as fast as *x* is increasing.
The equation of the tangent line is $y = f'(3)(x-3) + f(3) = 6(x-3)+9$.
I prefer to leave it in this form but webassign will probably want it in *y=mx+b* form, *y=6x-9*.

Remember that $\Delta$ means "the change in". $\Delta y = m\Delta x$ for a line and $\Delta y \approx f'(a)\Delta x$ for a curve. For the example above, if $\Delta x = 0.1, \quad \Delta y \approx 0.6$ This means that $f(3.1)-f(3)$ is about 0.6 so the square of 3.1 is about 9.6.
It is actually 9.61.
9.6 is the y-value on the tangent line when $x$ is 3.1.
Exercise: Let $\Delta x = -0.1, \quad \Delta y \approx -0.6$ and compare the value of the tangent line at $x$=2.9 to $f(2.9)$

Linear property of the derivative:
If A and B are numbers and f '(a) and g'(a) both exist then
(Af + Bg)' (a)=Af '(a)+Bg '(a).

Example: Using examples above, if $h(x) = 4x^2 + 5x - 10 \quad h'(3) = 4(6) + 5 - 0 = 29$ .

Just as the sign of the slope of a line tells whether the line is rising or falling as x moves to the right, the derivative tells whether or not the graph is rising or falling at a as x moves to the right of a. We know that $f(x) = x^2$ is rising to the right of 0 and falling to the left of 0. $f'(3)$=6 is positive because $f$ is increasing as $x$ moves to the right of 3. Find $f'(-3)$ and you will see it is negative, as $f$ is decreasing as $x$ moves to the right of -3.