Math 131 Section 3.4 The Chain Rule , finding the derivative of a composition.

Given $f(u)=2u+4$ and $u(x)=-3x+2$, we know a change of 1 in x will change $u(x)$ by -3=slope of $u(x)$.
A change of 1 in u will change $f(u)$ by 2= slope of $f(u)$ and in general $\Delta f=2\ \Delta u$.
So a change of 1 in x changes $u(x)$ by -3 which changes $f(u)$ by -6.
That is if $\Delta x=1$ then $\Delta u=-3$ and $\Delta f=2(-3)=-6$ In general $\frac{\Delta f}{\Delta x}=\frac{\Delta f}{\Delta u}\frac{\Delta u}{\Delta x}$

This is the basis for the chain rule. Now we will find $f((u(x))$ and the derivative of f with respect to x.
$f(u(x))=2u(x)+4=2(-3x+2)+4=-6x+8$. So we see the derivative of f with respect to x is $-6=2(-3)$.

The Chain Rule: $\frac{df}{dx}=\frac{df}{du}\frac{du}{dx}=f'(u(x))u'(x)$

Examples in which $u(x)=x^2+3$:

a) $h(x)=(x^2+3)^5 \quad h'(x)=5(x^2+3)^4(2x)$

Here $h(x)=f(u(x))$ where $f(u)=u^5$

$f'(u)u'(x)=5u^4(2x)=5(x^2+3)^4(2x)$

b) $h(x)=e^{x^2+3} \quad h'(x)=e^{x^2+3}(2x)=2xe^{x^2+3}$

c) $h(x)=\sin(x^2+3) \quad h'(x)=\cos(x^2+3)(2x)=2x\cos(x^2+3)$

Examples in which $u(x)=\frac{1}{x} \quad u'(x)=\frac{d}{dx}(x^{-1})=-x^{-2}=-\frac{1}{x^2}$

a) $h(x)=(\frac{1}{x}+3)^4 \quad h'(x)=4(\frac{1}{x}+3)^3(-\frac{1}{x^2})$

b) $h(x)=e^{\frac{1}{x}} \quad h'(x)=-\frac{1}{x^2}e^{\frac{1}{x}}$

c) $h(x)=\cos(\frac{1}{x}) \quad h'(x)=-\sin(\frac{1}{x})(-\frac{1}{x^2})=\frac{1}{x^2}\sin(\frac{1}{x})$

Examples in which $u(x) = \sec x \quad u'(x) = \sec x \tan x$

$a)\ h(x) = \sec^2 x \quad h'(x) = 2\sec x(\sec x \tan x) = 2\sec^2 x \tan x$

$b)\ h(x) = e^{\sec x} \ h'(x) = e^{\sec x} \sec x \tan x$